Earthquake Engineering
Analysis and Design

JULY 2011

MAVERICK UNITED CONSULTING ENGINEERS

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1.1 Earthquake Induced Vibrations

1.1.1 Engineering Seismology

1.1.1.1 Seismic Risk

Seismic Risk = Hazard x Exposure x Vulnerability x Specific Loss
(assess) (assess) (reduce) (reduce)

Hazard is the probability of a potentially damaging earthquake effect occurring at the site of a construction within its design life; It involves the level of seismic effect and the corresponding probability of occurrence. Hazard is to be assessed by seismic hazard assessment. Exposure is the concentration of human, commercial and industrial activity at a site subject to earthquake effects. Vulnerability is the level of damage that will be experienced by a structure when exposed to a particular earthquake effect; Vulnerability is to be reduced by sound structural earthquake engineering. Specific loss is the cost of restoring a structure to its original condition as a proportion of the cost of demolition and rebuilding a similar structure; Specific loss must be minimised by sound structural earthquake engineering. The Kobe earthquake of magnitude M_S = 7.0 on 17 January 1995 in Japan left 5,420 people without life and caused US$150 billion, the largest ever single loss from an earthquake.

1.1.1.2 Physics of the Earth

Earth's radius = 6,370 km so an arc on the surface of the planet subtending an angle of 1° at the centre has a length of 111km. Crust is composed of brittle granitic and basaltic rocks. Crust thickness = 10–15 km in oceanic areas and 30–50 km in continental areas. Lithosphere = Outer part of mantle and the crust; It has a thickness of about 60km in oceanic regions and 100km in continental regions; The lithosphere is the only part of the earth that exhibits brittle characteristics and hence the only part where earthquake can occur. Focal depths greater than the thickness of the lithosphere occur in the subduction zones where the lithosphere descends into the mantle.

1.1.1.3 Plate Tectonics

The convection currents in the asthenosphere (soft layer of about 200km in thickness below lithosphere) cause the 1-5cm/year movement of the lithosphere, which is divided into 12 major tectonic plates. The movement can be catagorised into: -

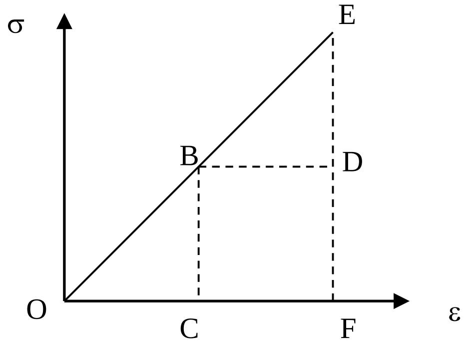
- (a) sea-floor spreading at mid-oceanic ridges as in the Atlantic Ocean
- (b) subduction as in the Pacific Coast of Central and South America
- (c) transcursion as along the San Andreas fault in California

1.1.1.4 Global Seismicity

The global distribution of earthquakes: -

- (a) Interplate
  - (i) circum-Pacific belt 75%
  - (ii) Alpide-Asiatic belt 22%
  - (iii) mid-oceanic ridges 2%
- (b) Intraplate
  - (i) Japan (significant tectonic deformation)
  - (ii) Australia, Northwest Europe, Brazil (cause not well understood)

1.1.1.5 Mechanism of Earthquakes: Elastic Rebound

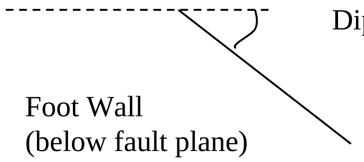


Path: -
OBD = Buildup of stresses
EB = Elastic Rebound

Energy: -
OEF = Energy in crust before earthquake
EBD = Stored elastic energy E_S released through elastic waves (P-waves and S-waves) in the elastic rebound mechanism
BDCF = Heat loss and inelastic deformation of the fault face
OBC = Remaining energy

1.1.1.6 Fault Rupture Classification

Elevation View

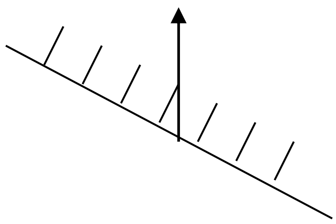


Dip, δ ($0° < \delta \leq 90°$)

Foot Wall
(below fault plane)

Hanging Wall
(above fault plane and at the side of the dip on plan)

Plan View



Strike, ϕ ($0° \leq \phi \leq 360°$)
(Clockwise from North measured to the fault dipping to the right of observer, i.e. the hanging wall on the right)
Here $\phi \approx 315°$.

Fault Plane View

Rake or slip, λ ($-180° \leq \lambda \leq +180°$)

- measured in plane of fault
- measured from strike line
- measured positive upwards

$\lambda = 0°$ (left-lateral)
$\lambda = \pm180°$ (right-lateral)
$\lambda = +90°$ (reverse)
$\lambda = -90°$ (normal)

Rupture Mechanism: -

(a) Horizontal strike-slip
  (i) Right-lateral
  (ii) Left-lateral
(b) Vertical dip-slip
  (i) Normal fault (tension, hanging wall below ground level)
  (ii) Reverse fault (compression, hanging wall above ground level)
(c) Oblique (dip and strike-slip)

1.1.1.7 Seismic Waves

<table>
<tr><td colspan="4">Seismic Waves</td></tr>
<tr><td colspan="2">Body Waves</td><td colspan="2">Surface Waves</td></tr>
<tr><td colspan="2">• Due directly to source of earthquake
• Faster than surface waves; Both V_P and V_S = function (elastic properties) where the stiffer the elastic medium the faster</td><td colspan="2">• The reflection of P- and S-waves at the surface back to the crust sets up a disturbance in the surface which then propagates along the surface; Since they are not radiated from the earthquake, they exist at some distance from hypocentre
• Slower than body waves; Both V_{LOVE} and $V_{RAYLEIGH}$ = function (elastic properties, wavelength)</td></tr>
<tr><td>P-Wave
(Primary Wave)</td><td>S-Wave
(Secondary Wave)</td><td>Love Wave</td><td>Rayleigh Wave</td></tr>
<tr><td>• Compression Wave
• Longitudinal Wave (particle motion parallel to propagation)
• $V_P > V_S$;
$V_P \sim 9$ km/s
$V_S \sim 5$ km/s
$V_P = \sqrt{\frac{E(1-\nu)}{\rho(1+\nu)(1-2\nu)}}$</td><td>• Shear Wave
• Transverse Wave (particle motion transverse to propagation)
• $V_P > V_S$;
$V_P \sim 9$ km/s
$V_S \sim 5$ km/s
$V_S = \sqrt{\frac{E}{2\rho(1+\nu)}} = \sqrt{\frac{G}{\rho}}$</td><td>• Horizontally polarised</td><td>• Vertically polarised like sea waves; but unlike sea waves, the particles under the waves travel in the opposite sense</td></tr>
</table>

1.1.1.8 Accelerographs and Seismographs

Characteristics	**Seismographs**	**Accelerographs**
Motion Recorded	Displacement / Velocity versus time Short period seismographs detect P- and S-waves Vertical long-period seismographs detect Rayleigh waves Horizontal long-period seismographs detect Love waves	Acceleration versus time Three components, namely 2 horizontal and 1 vertical records
Date of invention	1890	1932
Natural Period	1-20 seconds or longer (Short period and long-period)	< 0.05 seconds
Installation Location	Far-field	Near-field (close to earthquake)
Operation	Continuous (will pick up earthquake instantly)	Standby, triggered by shaking (will not pick up earthquake instantly)
Film Cartridge Speed	1 inch/minute	1 cm/second
Use of records	Seismology	Engineering

1.1.1.9 Earthquake Location Parameters

Parameters to define earthquake location: -

(i) Epicentre N, E ; Errors of the order of 5km

(ii) Focal depth, h_0 (km); Errors of the order 10km

(iii) Origin time t_0 (GMT)

1.1.1.10 Estimation of Location of Focus or Epicentre

(i) Local shallow earthquakes

(assuming constant velocity of waves, shallow depth i.e. $D_{FOCUS} \sim D_{EPICENTRE}$ & no earth curvature)

$$\text{Distance of hypocentre (focus) from station, } D_{FOCUS} = \left(\frac{V_P V_S}{V_P - V_S}\right)\delta T$$

Three stations' records of D_{FOCUS} are necessary to estimate the location of the focus or epicentre.

(ii) Non-local earthquakes

Use travel-time curves. By inputting the difference in travel times, δT between two phases, we can obtain the epicentral distance Δ° in degrees (x 111km to change to km) for any particular focal depth.

1.1.1.11 Estimation of Focal Depth

(i) From visual inspection of the seismogram, deep earthquakes have weak surface waves; Shallow waves are a clear indication of a shallow focus. For deep earthquakes use depth phases (pP, sS, etc)

(ii) If all else fails, assign arbitrary value of 10km or 33km

1.1.1.12 Estimation of Time of Earthquake Occurrence

Input: -

Absolute time that the first earthquake wave reaches station, AT
Distance from station to epicentre, D
Focal depth, h
Depth of topmost crust layer, d_1

Compute: -

Velocity at the topmost crust layer of d_1, V_1
Velocity at the second crust layer, V_2
Using Snell's Law, compute i_C

$$\frac{\sin i}{V_1} = \frac{\sin r}{V_2}, \text{hence} \frac{\sin i_C}{V_1} = \frac{\sin 90°}{V_2}$$

Output: -

The travel time from focus to station is the lesser time in the following phases: -

$$T_g = \frac{\sqrt{D^2 + h^2}}{V_1}$$

$$T^* = \frac{d_1}{V_1 \cos i_C} + \frac{(d_1 - h)}{V_1 \cos i_C} + \frac{D - (d_1 - h)\tan i_C - d_1 \tan i_C}{V_2}$$

Hence, the absolute time of earthquake = AT – travel time

1.1.1.13 Earthquake Magnitude

(a) Moment Magnitude, M_W based on Seismic Moment M_0

Proportional to the actual rupture slip displacement size of the earthquake, U(m)

Seismic Moment, M_0 = measures the size of an earthquake rupture
= work done in rupturing the fault
= μAU Nm (~ 10^{15} – 10^{30})

where the rigidity of the crust = $\mu \sim 3 \times 10^{10}$ (Nm^{-2})
displacement = U ~ 10^{-4} of rupture length of large earthquakes (m)
area = A (m^2)

Moment Magnitude, $M_W = (2/3)\log_{10}(M_0) - 10.7$

Wells and Coppersmith (1994) proposed an equation to estimate the earthquake potential of a fault of a given length, namely,

$$M_W = a + b\log(SRL)$$

where *SRL* is the surface rupture length in kilometers and

Fault type	*a*	*b*
Strike slip	5.16	1.12
Normal	4.86	1.32
Reverse	5.00	1.22
All	5.08	1.16

This based on the fact that rupture length grows exponentially with magnitude.

Note that the elastic energy released grows exponentially with magnitude

$$\log_{10}(E_S) = 11.8 + 1.5M_W$$

where E is measured in ergs. A unit increase in magnitude corresponds to a $10^{1.5}$ or 32-fold increase in seismic energy. Energy release from a M7 event is 1000 times greater than that from a M5 event. An observation is that E_S / Volume ~ constant.

(b) Richter's Local Magnitude, M_L recalibrating for specific area

Proportional to the S-P interval δt and maximum trace amplitude A

Richter's Local Magnitude, M_L from nomogram inputting δt and maximum trace amplitude A (mm), or Richter's Local Magnitude, $M_L = \log(A) - \log(A_0)$ of Richter's tables inputting maximum amplitudes A (mm) and epicentral distance from station, Δ (km).

(c) Teleseismic Magnitude Scales, M_S and m_b

Proportional to the maximum ratio of ground amplitude A (μm) to period of ground motion T (s) and hence independent of the type of seismograph

$$\text{Surface wave magnitude, } M_S = \log\left(\frac{A}{T}\right)_{MAX} + 1.66\log(\Delta°) + 3.3 \text{ (using long - period instruments)}$$

$$\text{Body wave magnitude, } m_b = \log\left(\frac{A}{T}\right)_{MAX} + Q(\Delta, h) \text{ (using short - period instruments)}$$

Moment Magnitude, M_W	Local Magnitude, M_L	Teleseismic Magnitudes M_S & m_b
Does not saturate at large earthquakes	Saturates at large earthquakes	Saturates at large earthquakes

Saturation refers to the fact that beyond a certain level, an increase in seismic energy release does not produce a corresponding increase in magnitude. This happens because the scales are based on readings of waves in a limited period range (determined by the characteristics of the instrument) and as the size of the earthquake source grows the additional energy release results in waves of larger period rather than increasing the amplitude of shorter period radiation.

Some major earthquakes of the 20th century are presented.

Year	Location	Richter Magnitude	Deaths
1970	Peru	7.6	70,000
1971	San Fernando, California	6.5	65
1976	Guatemala	7.5	23,000
1976	Tangshan	7.6	650,000
1976	Turkey	7.3	5,000
1976	Friuli, Italy	6.5	968
1977	Romania	7.2	2,000
1978	Miyagiken-Oki, Japan	7.4	27
1979	Montenegro, Yugoslavia	7.3	121
1985	Mexico City, Mexico	8.1	10,000
1989	Loma Prieta, California	7.1	62
1991	Northern India	7.1	768
1992	Turkey	6.8	800
1992	Landers, California	7.4	0
1992	Egypt	5.4	500
1993	Hokkaido, Japan	7.8	200
1993	Guam	8.1	0
1993	Maharashtra, India	6.4	12,000-30,000
1994	Northridge, California	6.7	33
1995	Kobe, Japan	7.2	5500
1999	Izmit, Turkey	7.4	17,400
1999	Chi-Chi, Taiwan	7.6	2,400
2001	Bhuj, India	7.9	25,000

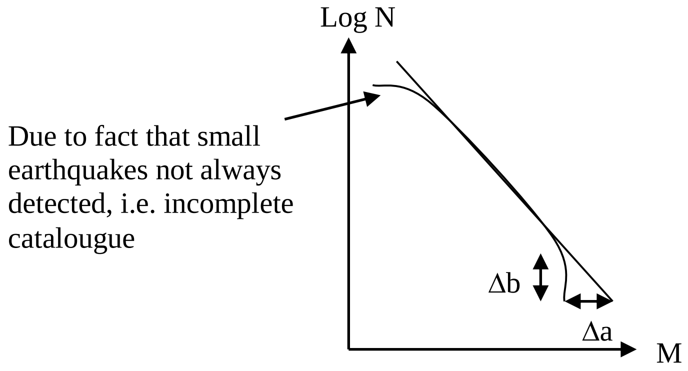
The 1989 Loma Prieta earthquake resulted in about 7 billion dollars in damage, not accounting for the loss of business opportunities. The total financial loss induced in the 1994 Northridge earthquake is estimated to be over 10 billion dollars.

1.1.1.14 Evaluation of Regional Seismicity

(a) The maximum credible earthquake M_{MAX} is first found from either
   (i) the maximum length of fault rupture that could physically occur
   $\log(L) = 0.7M_S - 3.24$ etc…
   (ii) $M_{MAX} = M_{MAX\ \text{KNOWN TO OCCUR}} + \Delta M$

(b) It is found that a linear relationship exists between frequency and magnitude,
   $\log(N) = a - b.M$ (Recurrence relationship by Gutenberg & Richter, 1954)
   N = number of earthquakes per year with magnitude ≥ M
   a = level of earthquake activity
   b = relative values of small and large earthquakes or brittleness of crust
   Putting $M = M_{MAX}$, we can compute N.

(c) Return Period, T = 1/N



Δa due to saturation of scale at high M
Δb due to period of observation being too short in comparison to the return period of large earthquake

1.1.1.15 Intensity

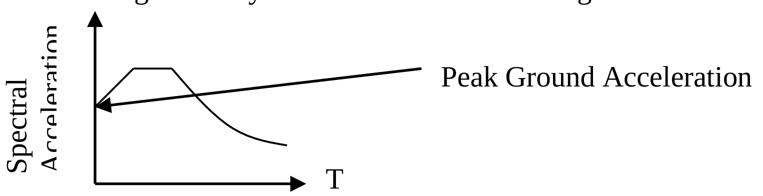
Intensity is an index related to the strength of the ground shaking and is not a measure parameter.

Intensity III → limit of perceptibility
Intensity VII → threshold of damage to structures
Intensity VIII → threshold of damage to engineered structures
Intensity X → realistic upper bound

Intensity worked out by taking the modal value of an intensity histogram.

1.1.1.16 Characterisation of Strong Motion

Horizontal and vertical Peak Ground Acceleration (PGA) is the most widely used parameter although it really represents the damage for only stiff structures since damage of a structure is also a function of its period.



Other parameters such as cycles of motion, frequency of motion and duration not taken into account.

PGA also does not correlate well with earthquake damage, except for stiff structures such as 1 storey brick buildings. The strong ground motion can also be characterised by the peak values of velocity and displacement obtained by the integration of the earthquake time-history. PGA correlates well with intensity; for damaging motion (intensity VIII), we find PGA ≥ 0.2g.

A widely used measure of damage potential is the energy in an accelerogram known as the Arias intensity.

$$I_A = \frac{\pi}{2g}\int_0^T a^2(t)dt$$

1.1.1.17 Attenuation Relationship

$$PGA = b_1 e^{b_2 M} d^{-b_3} e^{-b_4 d}$$

The coefficients are determined by regression analysis on a database of accelerograms' PGA and their associated magnitudes and distances.

PGV and PGD poor because the integration of acceleration time history heavily influenced by the errors associated with analogue accelograms and the processing procedures (filters) that are applied to the records to compensate for them. Also, the integration assumes initial values of zero for velocity and displacement although for analogue recordings, the first readings are lost and hence, this may not be the case. Small errors in baseline estimation of acceleration cause great errors in velocity and displacement.

1.1.1.18 Seismic Hazard Evaluation Procedure: Seismic Hazard Curves and Seismic Hazard Maps

The objective of seismic hazard assessment is to determine the probability of exceedance of a particular level of PGA at the site under consideration.

(a) Seismic hazard curves are produced for a particular point or site: -
- (i) Choose design life-span L for structure, say 50 years
- (ii) Determine seismic source points, I (seismic source lines and seismic source areas or zones could be converted to a number of seismic source points). Seismic source zones are delimited from catalogues of historical earthquakes.
- (iii) For each earthquake source i, determine M_{iMAX} and hence PGA_{iMAX}

  For each source i

  determine M_{iMAX} knowing $M_S \alpha$ log(rupture length) and catalogues of historical earthquakes

  using attenuation $\log(PGA) = c_1 + c_2M - c_3\log(d^2 + h_0^2)^{1/2} - c_4(d^2 + h_0^2)^{1/2}$ compute PGA_{iMAX}

  End For
- (iv) Determine LARGEST(PGA_{iMAX})
- (v) Choose a PGA ≤ LARGEST(PGA_{iMAX}) and compute M_i for each source

  For each source i

  using attenuation $\log(PGA) = c_1 + c_2M - c_3\log(d^2 + h_0^2)^{1/2} - c_4(d^2 + h_0^2)^{1/2}$ compute M_i

  End For

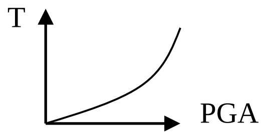
  Note that PGA of 0.2g corresponds to Intensity VIII that causes structural damage
- (vi) Compute annual frequencies N_i from recurrence relationships particular to each source

  For each source i

  $\log(N_i) = a - bM_i$, parameters obtained from catalogues of instrumental seismicity

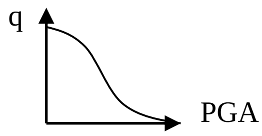
  End For

  If $M_i > M_{iMAX}$ then put $N_i = 0$



- (vii) Compute percentage of probability of exceedance, q

  $$q = 1 - e^{-L\sum_{i=1}^{i=i} N_i} \text{ (based on Poisson probability model)}$$



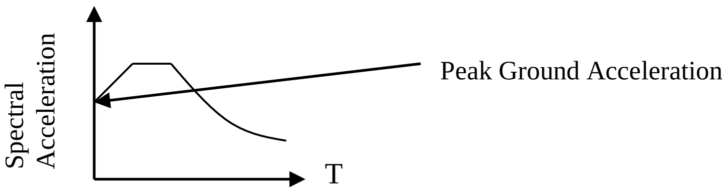
- (viii) Compute return period T

  $$T = 1 / \Sigma N$$
- (ix) Plot a point on the seismic hazard curves q vs PGA and T vs PGA
- (x) Loop to (iv) until complete plot of seismic hazard curves q vs PGA and T vs PGA is obtained

(b) Hazard map showing the contours of PGA for a site with a specific probability of exceedance q *or* a certain return period T is obtained by drawing a hazard curve at numerous grid points and reading off the PGA for a particular q *or* T. Remember that q, T, L and N are related by

$$q = 1 - e^{-L\sum_{i=1}^{i=i} N_i} \text{ (Poisson) and } T = 1/\Sigma N$$

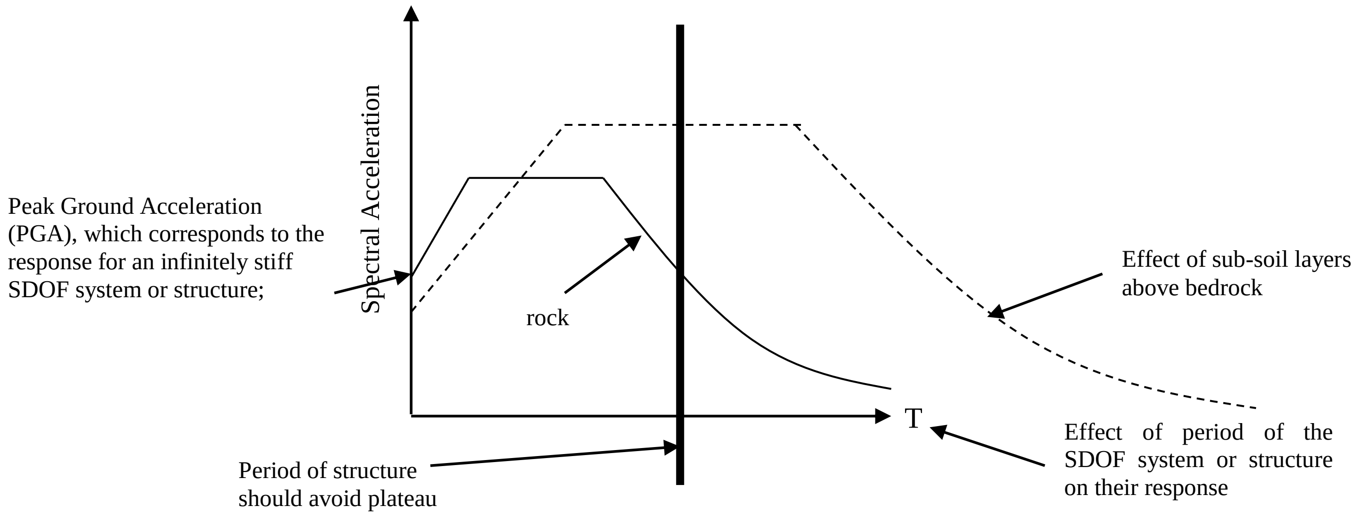
The Poisson distribution is stationary with time i.e. independent of time, hence the probability of earthquake occurring the year after is the same. Thus no elastic rebound theory inculcated. The hazard map allows the engineer to obtain a design PGA for a particular q *or* T. However, this value of PGA does not characterise the nature of strong motion and it is also of little use as an input to structural analysis except for an infinitely stiff structures. For general structures, we employ the design response spectra.



(c) Uncertainties in Seismic Hazard Assessment
- (i) Uncertain scatter in attenuation relationship
- (ii) Uncertain limits of source zone and hence the value of M_{MAX}
- (iii) Uncertain choice of attenuation relationship

1.1.1.19 Seismic Hazard Evaluation Procedure: Design Response Spectra

The response spectrum is a graph showing the maximum response of a range of SDOF systems, with a specific level of damping, subjected to a particular accelerogram (i.e. strong-motion). A SDOF is fully characterised by its damping level ζ and its natural period of vibration $T = 2\pi(m/k)^{-½}$. The response spectrum is plotted by calculating the response of a series of SDOF systems (with differing T but same ζ) subjected to a particular acceleration time-history at their base. Hence the response spectrum is a plot of relative displacement, relative velocity or absolute acceleration versus period T.



PGA is the response on an infinitely stiff SDOF system. The inclusion of the period of the SDOF system gives rise to the response spectrum. The effect of softer and less stiff (more flexible) sub-soil layer above the stiff bedrock changes the response spectrum in the following manners: -

(i) Amplitude of the response increases
Velocity of propagation is less in soil than in bedrock. In order to maintain the energy carried by the waves, the amplitude increases and the amount depends upon the contrast in propagation velocity in the soil and the bedrock, the softer the soil, the greater the amplification.

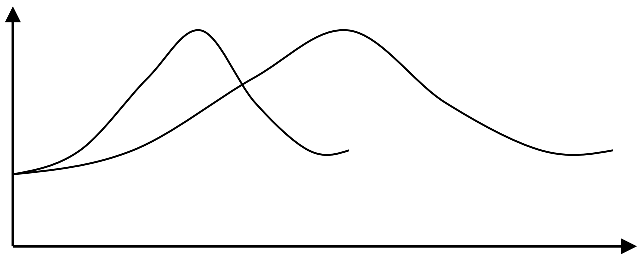
(ii) Amplitude of the spectrum increases dramatically if resonance occurs
If the dominant period of the ground motion (large earthquakes tend to produce dominant long-period waves whilst smaller earthquakes produce short period waves) i.e. $T_1 = 4H/S$ coincides with the natural period of vibration of the soil layer, then resonant response can result in very high amplitudes on the spectrum at that particular period. This happened in Mexico City on the 19th of September 1985 where a large M_S 8.1 earthquake caused greatest damage in the city underlain by soft lacustrine clay deposits although 400km away from the source.

(iii) Duration of strong motion increases
Waves can be reflected at the surface and then as they propagate downwards they can again be reflected back upwards at the rock face and in this way become trapped within the soil layer. This will have the effect of increasing the duration of the strong ground-motion.

(iv) Maximum amplitude occur at higher periods for soil spectra and at lower periods for rock spectra
This occurs because softer soil (less stiff, more flexible, lower frequency, higher period) tend to amplify high period waves and harder rock (more stiff, less flexible, higher frequency, lower period) tend to amplify low period waves.

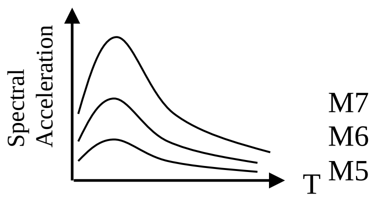
(v) PGA reduces
The value of the PGA is also dependent upon the sub-soil conditions (stiffness) although not usually taken into account in the attenuation relationship. A soft (flexible) soil will tend to reduce the value of the PGA as low period (high frequency) waves are filtered by the soft soil.

The earthquake magnitude M and the distance d change the response spectra in the following manner. Small nearby earthquake produce high frequency dominant waves. Large faraway earthquakes produce low frequency dominant waves.



The seismic hazard assessment procedure by the method of design response spectrum involves either: -

(a) PGA determination from hazard map and spectral shape fixation from codes of practice
PGA = corresponds to the response for an infinitely stiff SDOF system or structure
= obtained from the hazard map
= function of (M, distance d, return period T and probability of exceedance q) although also dependent upon the soil stiffness
Spectral Shape = corresponds to the response of a variety SDOF system or structure with different stiffnesses (or periods)
= function of (SDOF or structure period, soil stiffness) although also a function of the earthquake magnitude in that the peak amplitude varies with M



(b) Response spectral ordinates
The response spectrum can also be obtained from equations predicting the response spectral ordinates. Such equations are derived in exactly the same way as attenuation relationships for PGA for a number of different response periods.
Response spectral ordinate equation = function of (M, distance d, SDOF period T, soil stiffness)

The response spectra by including the PGA, the spectral shape and its amplitude, accounts for M, d, the structural periods T and soil stiffness. The response spectrum falls short in the sense that the duration of the earthquake not accounted for. Long duration earthquakes are obviously more damaging. Duration is a function of M, the larger the M, the longer the duration.

The larger the earthquake the longer the duration, the longer the period of the waves and the larger the spectral amplitudes.

1.1.2 Effect of Local Soil Conditions on the Seismic Hazard

1.1.2.1 Elastic Soil Properties

The elastic properties of soil are defined by the following 3 parameters: -

(a) the mass density, ρ

(b) the elastic moduli

(i) the shear wave velocity S or V_S is measured
S or V_S obtained from explosion tests, or
S or V_S = 85 $N_{60}^{0.17}$ $(D_m)^{0.2}$ (Seed 1986 empirical using SPT and CPT tests)

(ii) Shear modulus G is computed from the shear wave velocity S

$$S = \sqrt{\frac{\mu}{\rho}} = \sqrt{\frac{G}{\rho}}$$

(iii) Poisson's ratio ν known

(iv) Young's Modulus E computed

$$G = \frac{E}{2(1+\nu)}$$

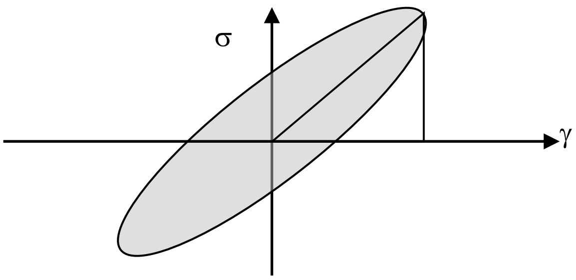
(v) Lame's constant λ computed; Note the other Lame's constant μ

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} \qquad \mu = G$$

(vi) compression wave velocity C is computed

$$C = \sqrt{\frac{\lambda + 2\mu}{\rho}}$$

(c) the damping



ΔW = area under loop
= energy lost in one loading-unloading-reloading cycle

W = area under the triangle

$$\text{Damping coefficient}, \zeta = \frac{1}{4\pi}\frac{\Delta W}{W}$$

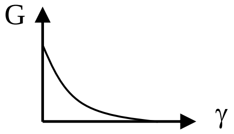
$$\text{Logarithmic decrement}, \delta = 2\pi\zeta$$

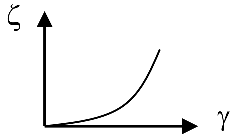
1.1.2.2 Dynamic Response of Soil

The dynamic response of soil depends upon: -

(a) whether the loading is small strain loading or large strain loading

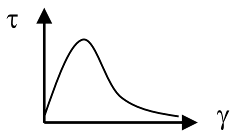
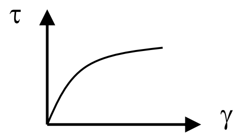
Earthquake loading may either be small strain or large strain. The shear modulus G and damping ζ of the soil depends on the strain level, hence non-linear behaviour. The shear modulus is represented by the secant modulus at any given strain.





Small Strain Loading	Large Strain Loading
High G, low ζ	Low G, high ζ
Soil can be modeled as viscoelastic material	Soil cannot be modeled as viscoelastic material as it is non-linear. Ground motion controlled by limit strength of soil. Hence the critical acceleration i.e. the limit acceleration the soil can transmit is used.
Field measurements of shear wave velocities generally give those at very low strain levels. This yields the maximum shear modulus G_{MAX}.	In general, G at any strain level can be related to G_{MAX} although the relationship differs from soil to soil.

(b) whether the loading is monotonic or cyclic

<table>
<tr><td>Dry soils or drained saturated soils</td><td colspan="2">Undrained saturated soils</td></tr>
<tr><td rowspan="3">No change in strength with cycles. Behaviour under cyclic loading same as that under monotonic loading.</td><td>Collapsible soil</td><td>Non-collapsible soil</td></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>Pore pressures rise continuously with cycles
Strength peaks and then reaches residual
Strain softening material</td><td>Pore pressures rise then tend to decrease
Strength increases
Strain hardening material</td></tr>
<tr><td></td><td>Stress (in the form of liquefaction) and strain is the criteria for failure</td><td>Strain is the criteria for failure, whilst stress failure (in the form of liquefaction) unlikely</td></tr>
<tr><td></td><td>Loose cohesionless sands
Normally consolidated clays</td><td>Dense sands
Overconsolidated clays</td></tr>
</table>

Note that the damping increases with cyclic loading.

(c) the frequency of the loading

(d) the speed of loading i.e. the rate of strain

Sand: the strain-rate has little effect on the strength

Clay: the strain-rate has considerable effect on the strength

(e) whether sliding occurs or not

If sliding occurs, strain is no longer applicable.

(f) the stress path

The undrained strength and the pore water pressures are stress path dependent and therefore limit strength of soil depend on how the failure state is arrived at. Since a soil element is subjected to 3 normal stresses and 3 shear stresses, the limit strength will depend on how each component is varied in the field during an earthquake.

1.1.2.3 Liquefaction

Liquefaction means failure due to rise on pore water pressures, and is not necessarily associated with a complete loss of strength. Hence, liquefaction is denoted by either of the following mechanisms: -

(a) ZERO EFFECTIVE STRESS IN LOOSE SOILS: Zero effective stress (and hence zero shear strength, i.e. complete loss of strength) occurs due to rise in pore water pressures; this occurs when the structure of loose saturated cohesionless soil collapses due to vibration and cyclic loading. When shear stress is applied under no volume change condition, some grains may lose contact with neighbouring grains, and therefore part of the load is taken up by the water. Thus excess pore water pressure is generated. On unloading, these grains do not go back to its original position and therefore, the excess pore water pressure remains. On further loading or reverse loading, more grains lose contact and more excess pore pressure is generated. Finally, after a number of cycles, if there is the possibility for all grains to separate from each other, then the entire load before cycling is taken up by the water and true liquefaction occurs. For this state to occur, the soil must be in very loose state at the beginning. After liquefaction, the liquefied soil will end up in a denser state in time through the dissipation of the excess pore pressure. In such a soil, collapse of the soil may take place even in static loading and shows very low residual strength.

(b) CYCLIC MOBILITY IN MEDIUM DENSE TO DENSE SOILS: Cyclic mobility i.e. the accumulation of large strains (or displacement along a shear surface) during cyclic loading is caused by the reduction of effective strength as a result of accumulation of pore water pressure. In defining cyclic mobility, there is thus no association with a complete loss of strength. However, the accumulation of strain may become very large and the soil may be considered as failed although the strength of the soil does not decrease as a whole even after failure.

ZERO EFFECTIVE STRESS IN LOOSE SOILS (Collapsible)	**CYCLIC MOBILITY IN MEDIUM DENSE TO DENSE SOILS (Non-Collapsible)**
Loose soils	Medium dense to dense soils
Low residual strength	Strength of soil does not decrease as a whole
Low strains	Large strains signifying failure

The cause of liquefaction in the field is the collapse of loose saturated cohesionless soils due to either: -

(a) cyclic loading, or
(b) static loading (quick sand conditions)

Manifestations of the liquefaction phenomenon include: -

(a) Bearing capacity failure in level ground causing tilting and sinking of structures
(b) Ground oscillation on very mild slopes, no lateral spreading but ground breaks into blocks which oscillate causing the opening and closing of fissures
(c) Lateral spreading of mild slopes with a cut or free slope due to the liquefaction of sub-surface deposits; ground breaks up causing large fissures and the ground generally moves slowly down the slope
(d) Flow failure of steep slopes due to liquefaction of the slope causing landslides, common in mine tailings
(e) Sand boils which are evidence of high pore water pressures at some depth due to liquefaction
(f) Rise of buried structures such as water tanks and timber piles which are lighter than the liquefied deposits
(g) General ground settlement occurring due to the densification of deposits after liquefaction
(h) Failure of quay walls (retaining walls) due to the increase of pressure on the wall as a result of liquefaction

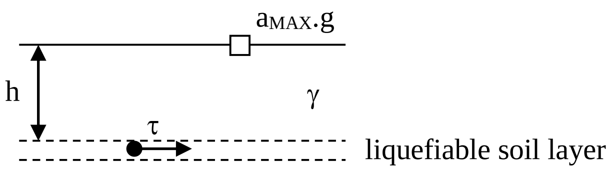
1.1.2.4 Residual Strength of Liquefied Soils

The residual strength of liquefied soil is an important parameter for estimating the stability of liquefied soils. The residual strength depends mainly on the void ratio but several other factors are noted, namely the initial confining pressure, the soil fabric, the fines content and the particle size and shape. The strength increases with increasing overconsolidation ratio. Increasing fines content decreases the steady strength at the same relative density.

1.1.2.5 Assessment of Liquefaction Potential

The liquefaction potential of a site depends on: -
(a) the size of the earthquake measured by M_S or M_W, preferably the latter
(b) the distance of the earthquake from the site
(c) the CRR values in terms of N_1^{60} or q_{c1N} or V_{S1} of the deposits
(d) the position of the water table with respect to the deposits

Liquefaction is most likely in loose saturated cohesionless sands. Gravelly soils being more permeable are unlikely to liquefy, although liquefaction of sandy gravel has occurred in the Kobe earthquake. But generally, the liquefaction resistance of sand-gravel composites increases considerably with increasing gravel content. Most clayey soils on the other hand are not vulnerable to liquefaction. However, laboratory studies indicate some loss of strength during cyclic loading and cyclic mobility may be significant. Increased plasticity generally increases the cyclic strength. The assessment of the liquefaction potential is as follows: -



(a) Cyclic stress ratio CSR
The CSR depends on the shear stress imposed by the earthquake as a function of the initial effective overburden pressure. This therefore depends primarily on the magnitude and distance of the earthquake, the depth of the liquefiable soil, the depth of the water table and the characteristics of the soil layer.

$CSR = \tau / \sigma' = 0.65\ (a_{MAX})\ (\gamma h/\sigma')\ r_d$

where h = depth to point of measurement or liquefiable soil

r_d = rigidity of soil to include the effect of more flexible soil at depth, hence lower shear stresses
= generally, r_d varies from 1 at the surface to 0.9 at 10m depth
= a function of the depth to the liquefiable layer, for instance 1-0.00765z (Liao & Whitman 1986)

γh = total overburden pressure at the level of the liquefiable layer

σ' = effective overburden pressure at the level of the liquefiable layer

a_{MAX} = peak ground acceleration at site from an attenuation relationship based on a maximum M_W

$\log(a_{MAX}) = -1.02 + 0.249M_{WMAX} - \log(r) - 0.0025r$ where $r^2 = R_f^2+53.3$ (Joyner & Boore 1981)

note that τ = mass/area x acceleration = $\gamma h/g$ x a_{peak}
note that $\gamma h/\sigma'$ takes the position of the water table into account
note that 0.65 of a_{MAX} represents 95% energy of the record
note that as with most attenuation relationships, a_{MAX} is the numerical fraction of g

(b) Cyclic resistance ratio CRR
The CRR depends on the quality of the soil and the depth of the liquefiable layer. This is defined as the cyclic stress required to cause liquefaction in a given number of cycles expressed as a fraction of the initial effective confining pressure. CRR is the required CSR for liquefaction. The CRR values are usually determined for liquefaction at 15 cycles of loading representing a 7.5 magnitude earthquake. Since the number of cycles is a function of the duration of the earthquake, which in turn depends on the magnitude, CRR is dependent on the magnitude of the earthquake as well. The factors effecting the CRR are void ratio and relative density, overconsolidation ratio, fines content and plasticity of fines. CRR can be determined from shear box and triaxial tests, but field measurements are preferable. CRR can be determined in the field by either one of the following methods: -

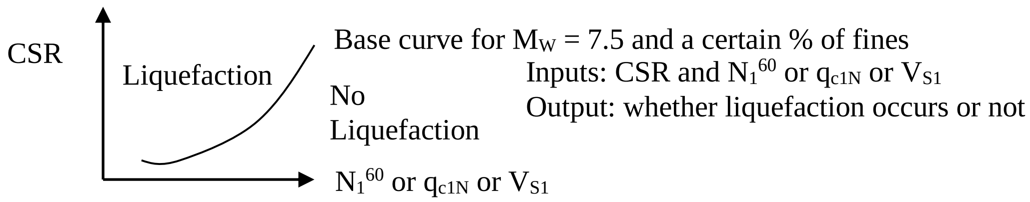
(i) CRR from SPT (standard penetration test) blow count N procedure
Normalised SPT value to depth and energy efficiency $N_1^{60} = N.C_N\ E / 60$
where N = measured blow count

C_N = depth normalisation factor = $(100/\sigma')^{0.5}$ or $0.77\log_{10}(2145/\sigma')$
σ' = effective stress at the point of measurement i.e. at the depth of the liquefiable soil (kPa)
E = energy efficiency of the SPT (%)
CRR = function of (N_1^{60}, M_W, fines content) obtained through field observation of liquefaction

(ii) CRR from CPT (cone penetration test) tip resistance q_c procedure
CPT is better than SPT in the sense that it provides a continuous reading unlike SPT which are taken at intervals. Also, repeatability with SPT tests is poor.
Normalised CPT value $q_{c1N} = q_c (100 / \sigma')^{0.5} / 100$
where q_c = measured tip resistance (kPa)
σ' = effective stress at the point of measurement i.e. at the depth of the liquefiable soil (kPa)
CRR = function of (q_{c1N}, M_W, fines content)

(iii) Shear wave velocity procedure
The use of S or V_S is advantageous because this can be measured easily with accuracy in soils in which SPT and CPT are difficult to perform. The disadvantage is that it is measured in situ with very small strains. Moreover, seismic testing is done without extracting samples and as such, difficult to identify non-liquefiable low velocity layers (soft clay rich soils) or liquefiable high velocity layers (weakly cemented soils). However, shear wave velocity measurements along with bore-hole data becomes very usable.
Normalised shear wave velocity $V_{S1} = V_S (100 / \sigma')^{0.25}$
where V_S = measured shear wave velocity
σ' = effective stress at the point of measurement i.e. at the depth of the liquefiable soil (kPa)
CRR = function of (V_{S1}, M_W, fines content)

(c) Factor of safety against liquefaction
F_L = CRR / CSR
Note that if F_L > 1.0, then no liquefaction.



Note that, for the same N value, higher percentage of fines shows higher CRR.

There is good agreement between the SPT and CPT procedures. Beyond $N_1^{60} > 30$ or $q_{c1N} > 160$, according to the NCEER (1997), liquefaction is very unlikely to occur. The base curves are plotted for a M_W of 7.5. For other magnitudes, scaling factors (MSF) are proposed which multiply the CRR base curves.

The reason that large magnitude earthquakes may liquefy sites at large distances even though the corresponding acceleration is small is due to the longer duration and therefore to the larger number of cycles. Liquefaction can be achieved by smaller number of cycles with large stress amplitudes and by larger number of cycles with smaller stress amplitudes.

1.1.2.6 Post-Seismic Failure due to Liquefaction

The consolidation of a liquefiable soil at a depth underneath a competent soil with time may increase the pore pressures in the top layer causing bearing capacity failure some time after the earthquake. The upward flow may even cause piping failure. The delay depends on the relative consolidation and swelling properties of the two layers.

1.1.2.7 Methods of Improving Liquefiable Soils

(a) Remove and replace unsatisfactory material (i.e. loose saturated cohesionless sands)
(b) Densify the loose deposits
(c) Improve material by mixing additives
(d) Grouting or chemical stabilisation
(e) Draining solutions

1.1.2.8 Effect of Soil Layer on Ground Response

The effect of local soil layer is as follows: -
(a) Impedance (or radiation) effect, hence increase in amplitude of strong motion
(b) Increase in duration of strong motion as energy becomes trapped within soil layer
(c) Resonance effect, hence a great increase in amplitude of strong motion

We consider vertically propagating polarised shear wave in energy transfer because: -
(a) due to the continuously decreasing stiffness of the soil or rock material, the wave front appears to travel vertically; an elastic wave propagating to a less stiff material (such as from stiffer rock to soil) will begin to propagate slower, since $V_S = (G/\rho)^{0.5}$ i.e. $V_S \propto G^{0.5} \propto E^{0.5}$ and $\rho_{rock} \sim \rho_{soil}$. A wave propagating slower will tend to move towards the normal to the interface between the 2 mediums.
(b) at a reasonable distance form the focus, these spherically propagating waves may be considered as plane propagating waves in a narrow wave front
(c) the energy carried by the shear waves is more important than that carried by the compression waves, these two waves being the only two emitted from the focus

1.1.2.9 Motion of Shear Waves in Elastic Media

The motion of shear waves in elastic an elastic medium is given by the equation of motion

$$\frac{\partial^2 u}{\partial t^2} = s^2\left[\frac{\partial^2 u}{\partial y^2} + \eta\frac{\partial^2 u}{\partial y^2}\right]$$

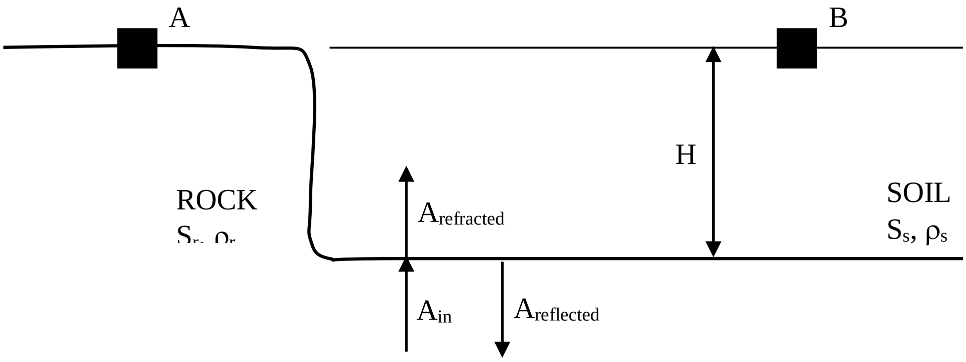
for which the solution is

$$u = Ae^{-\lambda\Omega s/S}e^{i\Omega(y/S-t)}$$

$$\begin{aligned}
\text{where } s &= \text{distance travelled within damped soil (m)}\\
y &= \text{vertical displacement (m)}\\
t &= \text{time (s)}\\
S &= \text{shear wave velocity (ms}^{-1}\text{)}\\
\Omega &= \text{circular frequency of the wave (rad/s)} = 2\pi f = 2\pi / T\\
\lambda &= \eta\Omega/2 \text{ (damping as a fraction of critical)}\\
Ae^{-\lambda\Omega s/S} &= \text{decaying amplitude term suggesting higher frequency waves decay faster}\\
e^{i\Omega(y/S-t)} &= \text{harmonic term, i.e. same response shown periodically}
\end{aligned}$$

1.1.2.10 Impedance (or Radiation) Effect

The impedance effect occurs because of the contrast of properties at the rock-soil interface.



(a) Impedance relationships

$$A_{in} = \text{Amplitude of incoming wave}$$

$$A_{refracted} = \frac{2}{1+r}A_{in} \ \ \& \ \ A_{reflected} = \frac{1-r}{1+r}A_{in} \quad \text{where } r = \frac{S_s \rho_s}{S_r \rho_r}$$

(b) Amplitude of concern, $A_{boundary}$

The wave motion at the boundary is the same whether one looks at it from the first or the second medium.

$$A_{boundary} = A_{refracted}$$

$$= A_{in} + A_{reflection}$$

$$\text{For } r = 0 : \text{free boundary with } A_{boundary} = 2A_{in}$$

$$r < 1 : \text{rock to soil with } A_{refracted} > A_{in}$$

$$r > 1 : \text{soil to rock with } A_{refracted} < A_{in}$$

$$r = 1 : \text{no boundary with } A_{refracted} = A_{in}$$

(c) Typical values

$$S_s = 300\text{ms}^{-1}, S_r = 3000\text{ms}^{-1}, \rho_s \approx \rho_r,$$

$$\therefore r = 0.1, A_{refracted} = 1.82A_{in} \ \& \ A_{reflected} = 0.82A_{in}$$

Notice that the soil has amplified the motion of the waves as they were impeded.

(d) Computation procedure for first half-cycle of strong-motion at site $A_{SOIL@B}$

$$\text{INPUT} : A_{ROCK@A} \text{ is an } A_{boundary} \text{ obtained from the attenuation relationship from a seismic hazard analysis}$$

$$\text{METHOD} : A_{in} = \text{Amplitude of incoming wave} = A_{ROCK@A} / 2 \text{ as } A_{boundary} = 2A_{in}$$

$$: A_{refracted} = \frac{2}{1+r}A_{in} \quad \text{where } r = \frac{S_s \rho_s}{S_r \rho_r}$$

$$\text{OUTPUT} : A_{SOIL@B} = 2A_{refracted} e^{-\lambda\Omega s/S} \text{ as } A_{boundary} = 2A_{in}$$

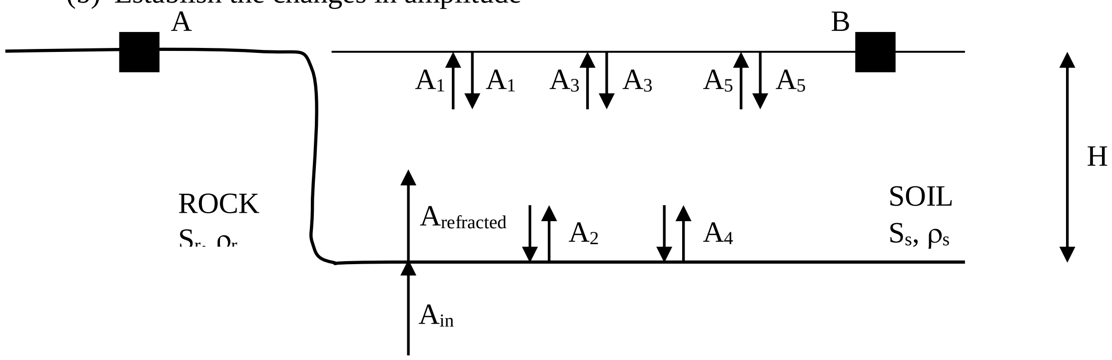
On the soil spectra, the effect of impedance is as follows: -
(a) Amplitude of the response increases
The effect of impedance is to increase the amplitude of the strong motion response. Velocity of propagation is less in soil than in bedrock. In order to maintain the energy carried by the waves, the amplitude increases and the amount depends upon the contrast in propagation velocity in the soil and the bedrock, the softer the soil, the greater the amplification. The energy flux of the incident wave is shared between the refracted and reflected waves.

1.1.2.11 Increase in Duration of Strong Motion

Waves can be reflected at the surface and then as they propagate downwards they can again be reflected back upwards at the rock face and in this way become trapped within the soil layer. This will have the effect of increasing the duration of the strong ground-motion. Computation of strong motion response with time: -

(a) Establish the input
Duration of earthquake wave D (s)
Fundamental period of earthquake wave T (s)
Damping of soil ζ
Relative property of soil-rock $r = S_s\rho_s / S_r\rho_r$
Relative property of rock-soil $\underline{r} = S_r\rho_r / S_s\rho_s$

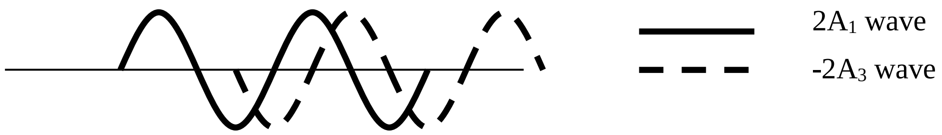
(b) Establish the changes in amplitude



(i) $A_{in} = A_{ROCK} / 2$
(ii) $A_{refracted} = 2A_{in}/(1+r)$ @ t=0 for duration D
(iii) $A_1 = A_{refracted}\ e^{-\zeta\Omega s/S}$ @ t=H/S for duration D
(iv) $A_2 = (1-\underline{r})/(1+\underline{r})\ A_1\ e^{-\zeta\Omega s/S}$ @ t=2H/S for duration D
(v) $A_3 = A_2\ e^{-\zeta\Omega s/S}$ @ t=3H/S for duration D
(vi) $A_4 = (1-\underline{r})/(1+\underline{r})\ A_3\ e^{-\zeta\Omega s/S}$ @ t=4H/S for duration D
(vii) etc…$A_{infinity}$ until negligible amplitudes

(c) Superpose waves at the top of soil layer for an estimate of a measured record
A sinusoidal wave of $2A_1$ starting @ t=H/S for duration D
A sinusoidal wave of $-2A_3$ starting @ t=3H/S for duration D
A sinusoidal wave of $2A_5$ starting @ t=5H/S for duration D
A sinusoidal wave of $-2A_7$ starting @ t=7H/S for duration D
etc…

All the individual waves have the same period and duration. Their amplitudes are doubled because of the free surface boundary. Every other wave has an opposite initial amplitude because when the waves get reflected at the bottom of the soil against the rock, there is a 180 degree phase change. These superposed waves will result in a summation wave that is increasing to a peak then decaying gradually.



(d) Check if resonance is likely to occur

Evidently, if the period of the initial earthquake incident wave is 4H/S, this would correspond to one loop cycle and resonance would occur. Also, resonance can occur if the earthquake wave period corresponds to the higher mode periods of the soil. In summary, if the earthquake period is any of the following, then resonance can occur.

Mode periods of the soil $T_n = T_1 / (2n-1)$, where n = 1,2,3,4… and T_1 = fundamental mode period

On the soil spectra, the duration of strong ground motion is increased.

1.1.2.12 Resonance Effect

The fundamental mode of period of the soil layer, $T = 4H/S_s$. Hence, the period along with the damping coefficient ζ give the layer characteristics. Thus for the same stiffness of material, the deeper the layer, the longer the period. Alternatively, for the same depth of layer, the weaker the layer, the longer the period.

Earthquakes that can affect an engineering site: -

(a) Small earthquakes nearby (near source)
Bedrock motion is rich in high frequency waves (low period)
Shallow H and stiff (high S) subsoil (low period) will magnify the bedrock motion in resonance
Deep H and weak (low S) subsoil (high period) will not magnify the bedrock motion in resonance

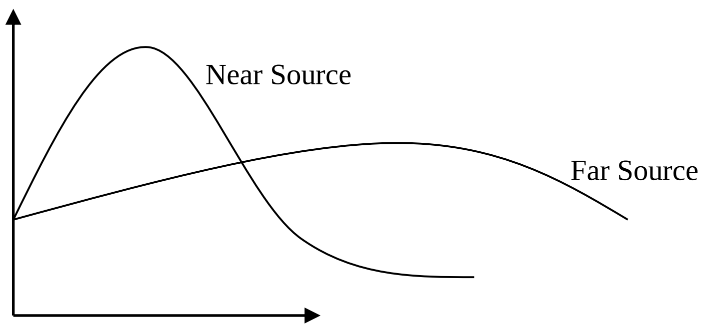
(b) Large earthquakes afar (far source)
Bedrock motion is rich in low frequency waves (high period) because high frequency waves decay fast
Shallow H and stiff (high S) subsoil (low period) will not magnify the bedrock motion in resonance
Deep H and weak (low S) subsoil (high period) will magnify the bedrock motion in resonance

On the soil spectra, the effect of resonance is as follows: -

(a) Maximum amplitude occur at higher periods for soil spectra and at lower periods for rock spectra
This occurs because softer soil (less stiff, more flexible, lower frequency, higher period) tend to amplify high period waves and harder rock (more stiff, less flexible, higher frequency, lower period) tend to amplify low period waves.

(b) PGA reduces
The value of the PGA is also dependent upon the sub-soil conditions (stiffness) although not usually taken into account in the attenuation relationship. A soft (flexible) soil will tend to reduce the value of the PGA as low period (high frequency) waves are filtered by the soft soil.

(c) Amplitude of the spectrum increases dramatically if resonance occurs
If the dominant period of the ground motion (large earthquakes tend to produce dominant long-period waves whilst smaller earthquakes produce short period waves) coincides with the natural period of vibration of the soil layer (i.e. $4H/S_s$), then resonant response can result in very high amplitudes on the spectrum at that particular period. This happened in Mexico City on the 19th of September 1985 where a large M_S 8.1 earthquake caused greatest damage in the city underlain by soft lacustrine clay deposits although 400km away from the source.



Response Spectra due to Near Source and Far Source Events

1.1.2.13 Methods of Evaluating Layer Response

Three methods can be used, namely: -

(a) The travelling wave method
This has been described in the Increase in Duration of Strong Motion section

(b) The Square Root of Sum of Squares (SRSS) method
Characteristics: -
Infinitely stiff rock base
Provides steady state and transient state solutions
Only good to find maximum acceleration A_{MAX}, i.e. no time history response
Damping coefficient refers to the modal damping

If we treat soil as an elastic material, then a column of soil over the rock (assumed rigid since $S_r >> S_s$) behaves like any other structure which has natural modes and mode shapes. The solution to such a problem is

$$\ddot{u}(y,t) = \sum_{n=1}^{\infty} \Phi(y,n) I(t,n)$$

$\Phi(y,n)$ = mode shapes depending on geometry and boundary conditions

$I(t,n)$ = response of SDOF systems of mode frequencies subjected to the applied load

Thus, a soil layer (or any elastic structure resting on a rigid base) is equivalent to a series of SDOF pendulums (characterised by their ω and λ) standing on a single rigid base. The total response of the soil layer is obtained by combining all the responses of each SDOF system.

$$\text{Peak acceleration, } \ddot{u}(y)_{MAX} = \sqrt{\sum_{n=1}^{\sim 4} \Phi^2(y,n) S_a^2(n)}$$

S_a is the equivalent acceleration spectrum values of the acceleration record. Note only 4 modes are used in the SRSS method. Note also that y=0 at the free surface and is measure positive downward.

INPUT: soil damping ζ, forcing frequency Ω (rad/s), soil layer H (m), soil shear velocity S_s (ms^{-1})

n	Mode shapes, ϕ	Period of system $T = 4H / [(2n-1)S]$	Frequency of system, $\omega = 2\pi/T$	S_a = function (ζ, Ω, ω)	ϕ^2	S_a^2	ϕ^2 x S_a^2
1							
.							
.							
4							

$A_{MAX} = \text{SQRT}(\Sigma_{n=1-4}\ \phi^2 \text{ x } S_a^2)$

(c) The SHAKE method
Characteristics: -
Base rock is considered flexible, hence radiation damping i.e. waves that travel into the rock from the soil, is taken into account
Provides only steady state solution
Gives time history response
Different layers can have different damping coefficients

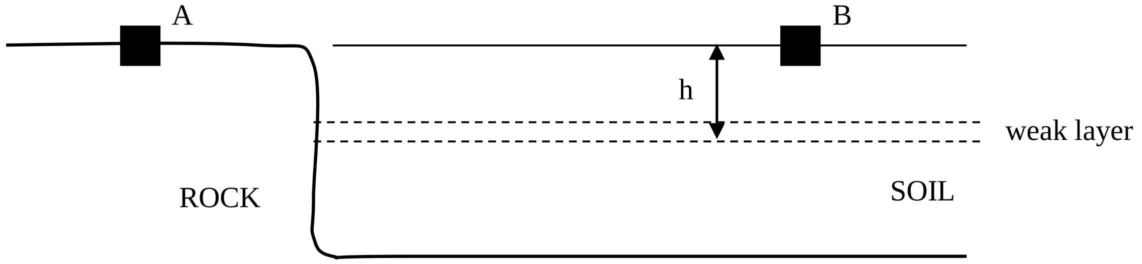
In this technique, a single harmonic component of the wave can be transferred from any known point to any other desired point by the use of a complex transfer function. By using Fourier Series of the known motion, each component of the series can be transferred to the desired point and by using the superposition technique, these component waves can be recombined to produce the time history of the motion at the desired point.

Hence, the SHAKE method is summarised as follows: -

I. input is the ground motion f(t)
II. f(t) is decomposed into multiple frequency Ω waves each having an amplitude A and phase θ
III. for each component wave Ω,
   the transfer function of amplitude T and of phase ϕ is computed
   output wave component amplitude is TA and phase is $\theta + \phi$
   next component wave
IV. the output wave components are superimposed for a time history response

1.1.2.14 Critical Acceleration

Small Strain Loading	Large Strain Loading
Soil can be modeled as viscoelastic material	Soil cannot be modeled as viscoelastic material as it is non-linear. Ground motion controlled by limit strength of soil. Hence the critical acceleration i.e. the limit acceleration the soil can transmit is used.



(a) Limit undrained strength of weak layer, c_u

$$\Delta u = B\left[\Delta\sigma_3 + A\left(\Delta\sigma_1 - \Delta\sigma_3\right)\right] \text{ (Skempton's)}$$

$$\text{Undrained strength, } c_u = \frac{c'\cos\phi' + \sigma'\left[K_0 + A(1-K_0)\right]\sin\phi'}{1-(1-2A)\sin\phi'}$$

Note that the change of pore water pressure Δu is dependent upon the change of principal stresses due to an earthquake. For saturated soils, B = 1. Under cyclic loading the pore pressure parameter A changes with cycles. The effect of rotation of the principal stresses is not taken into account. The undrained strength depends upon the change of pore water pressures. Hence, both Δu and c_u are dependent upon the number of cycles.

(b) Applied stress at weak layer, τ

Applied stress, τ = force/area x acceleration
= m/A x a = $\rho Va/A = \rho hAa/A = \rho ha$
= $\gamma ha/g$ (i.e. total stress / g x acceleration)
= $\gamma hk_m g/g$ as $a = k_m g$
= γhk_m

(c) Critical acceleration

When $\tau = c_u$ then the acceleration is critical

For c' = 0,

$$\tau = c_u$$
$$\gamma hk_c = \sigma_1'\alpha \qquad \alpha = \frac{\left[K_0 + A(1-K_0)\right]\sin\phi'}{1-(1-2A)\sin\phi'}$$
$$k_c = \sigma_1'\alpha/\gamma h$$
$$= \alpha\, \sigma_1'/\sigma_1$$

For c'=0, and say $\phi'=30°$, $K_0=0.6$, $\sin\phi'=0.5$, A=0.4 and with water table at ground level, $k_c = 0.42\gamma'/\gamma$ = 0.2. This means that, the weak soil layer will fail if a=0.2g.

(d) Consequences

If the elastic response predicted a magnified acceleration at B of say 0.5g from an incoming acceleration of 0.2g at A, this non-linear critical acceleration analysis will tell us that in actual fact the response at B would be truncated to 0.2g. However, the upper part of the soil will behave like a sliding block trying to move over the sliding surface and we may see ground cracks on the surface. This sliding block displacement is predicted by

$\log(u_{max})=f(k_c/k_m)$ where $k_c g$ = critical acceleration $k_m g$ = maximum elastic acceleration

such as $\log(4u_{max}/Ck_m gT^2)=1.07-3.83k_c/k_m$ (Sarma 1988);

Hence,

(i) if elastic response k_m < critical k_c, k_m applies and no sliding block displacement
(ii) if elastic response k_m > critical k_c, k_c applies and there is a sliding block displacement.

1.1.2.15 Effect of Local Soil Conditions on Seismic Hazard Design Procedure Summary

The objectives of the analysis are: -

(a) to provide a soil surface acceleration response taking into account the properties of the soil layers overlying the rock. The peak ground acceleration of the free-field rock is 0.6g from the previous seismic hazard assessment of **Section 1.1.1.18**.

(b) to assess the liquefaction hazard of the site

1.1.2.15.1 Soil Surface Acceleration Response

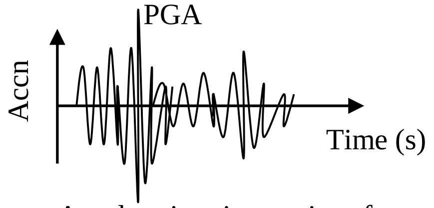
(a) Design Rock PGA from the Seismic Hazard Analysis

(b) Design Free-Field Rock Acceleration Time History

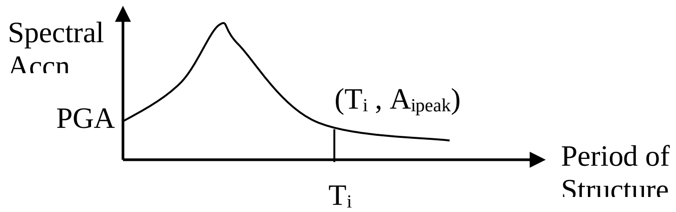
Plot of acceleration (g) with respect to time as measured on the free-field rock scaled to the design PGA from the hazard analysis of (a).

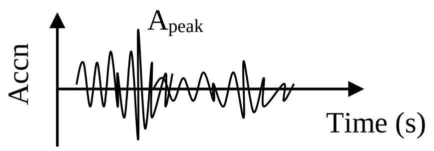
(c) Acceleration Spectrum of the Free-Field Record

Plot of spectral acceleration (maximum acceleration) with respect to the period of the structure. This response spectrum shows the maximum response (in terms of acceleration) of a range of single degree of freedom systems with a specified level of damping subjected to the above free field rock acceleration time series. The PGA is the intersection at the vertical axis and its value is the maximum observed on the acceleration time series of (b), i.e. the response of an infinitely stiff structure. In short: -



Acceleration time series of an infinitely stiff structure





Acceleration time series of an finitely stiff structure of period T_i

Alternatively, the rock acceleration spectrum can be obtained from the fixation of a spectrum shape (from the design codes) to the design rock PGA of (a). Thirdly, the rock acceleration spectrum can also be obtained from spectral ordinates.

(d) Fourier Spectrum of the Rock Record

Plot of modulus with respect to frequency (or period) of the earthquake wave derived from the time series of (b). The Fourier spectrum is a mathematical representation of the infinite wave harmonics that make up the earthquake wave record time series of (b). That is to say, the earthquake record can be broken up into an infinite number of waves defined by an amplitude and frequency. The Fourier Spectrum of the rock record is thus the mathematical representation of the earthquake wave harmonics prior to modification due to the soil layers. Two graphs define the Fourier spectrum, namely the modulus versus frequency graph and the phase versus frequency graph. The frequency at which the maximum modulus occurs is the dominant frequency of the earthquake wave time series prior to modification by the soil layers.

$$\text{Fourier Spectrum} = \sum a_n e^{i(\omega_n t - \phi)}$$

(e) Shear Wave Velocity Profile

Plot of shear wave velocity with respect to depth. Shear wave velocity, V_S (m/s) = 85 $N_{60}^{0.17} (D_m)^{0.2}$ i.e. based on the SPT. However it is best to carry out proper seismic blast tests.

(f) Transfer Function

Plot of modulus with respect to frequency of the waves derived from the shear wave velocity profile of (e).

(g) Fourier Spectrum of the Soil Surface

Plot of modulus with respect to frequency (or period) of the earthquake wave. The Fourier Spectrum of the soil surface is the mathematical representation of the earthquake wave harmonics after the modification due to the soil layers. These are obtained by multiplying the Fourier spectrum of the rock record by the transfer functions. The dominant frequency of the earthquake waves are usually lower in the soil modified Fourier spectrum compared with the rock Fourier spectrum. This is expected as the more flexible (high natural circular period) soil layers tend to lower the frequency (or increase the period) of the earthquake waves.

(h) Design Soil Surface Acceleration Time History

Plot of acceleration (g) with respect to time obtained from the Fourier spectrum of the soil surface of (g).

(i) Acceleration Spectrum at the Soil Surface

Plot of spectral acceleration (maximum acceleration) with respect to the period of the structure. This response spectrum shows the maximum response (in terms of acceleration) of a range of single degree of freedom systems with a specified level of damping subjected to the above soil surface acceleration time series of (h). The PGA is the intersection at the vertical axis and its value is the maximum observed on the acceleration time series of (h), i.e. the response of an infinitely stiff structure.

(j) Maximum Shear Stress Profile & Shear Strength Profile

The maximum shear stress $\tau_{applied}$ profile (plot of shear stress with respect to depth) is obtained from the accelerations at each level of the profile. The maximum shear strength $\tau_{capacity}$ profile is obtained from c_u

$$\tau_{applied} = c_u = \frac{c'\cos\phi' + \sigma'\left[K_0 + A(1-K_0)\right]\sin\phi'}{1-(1-2A)\sin\phi'}$$

Wherever $\tau_{applied} > \tau_{capacity}$, the critical acceleration has been exceeded.

(k) Applied Acceleration Profile & Critical Acceleration Profile

Applied acceleration $k_m = \tau_{applied} / \gamma h$ and critical acceleration $k_c = \tau_{capacity} / \gamma h$.

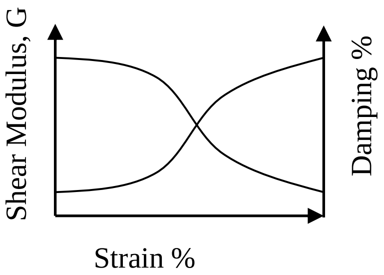
(l) Soil Surface Design Acceleration

Define the minimum critical acceleration as the smallest critical acceleration within the soil profile considered. Hence, the soil surface design acceleration is the smaller of: -

(a) the minimum critical acceleration k_{cmin}, and

(b) the elastic response at the soil surface, which is given for different periods (of the SDOF structures), in the response spectrum of (i)

(m) Effect of Large Strains on Shear Velocity



The shear modulus, G decreases with strain. Since shear wave velocity, S is proportional to $G^{0.5}$, S will also decrease with large strains. When there is large strain, soil particles are not as tightly packed, thus the shear wave cannot travel as quickly.

The reduction in shear velocity will cause a slight reduction in the dominant frequency of the earthquake waves because of the increase flexibility of the soil layers. However, the consequences of this are minimal.

(m) Sliding Block Displacement

Sliding block displacement occurs whenever $k_m > k_c$

This sliding block displacement is predicted by

$\log(u_{max})=f(k_c/k_m)$ where $k_c g$ = critical acceleration $k_m g$ = maximum elastic acceleration

such as $\log(4u_{max}/Ck_m gT^2)=1.07-3.83k_c/k_m$ (Sarma 1988);

1.1.2.15.2 Liquefaction Hazard Analysis

		Cyclic Stress Ratio CSR					Cyclic Resistance Ratio CRR			
Layer No	Depth (m)	Shear Stress, $\tau = k_m\gamma h$	Soil rigidity, r_d	Unit Weight (kN/m3)	σ'	CSR = $0.65\tau\, r_d / \sigma'$	N_{60}	Depth normalisation factor, $C_N = (100/\sigma')^{0.5}$	$N_1^{60} = C_N N_{60}$	Liquefaction if CSR>CRR

The liquefaction hazard is ascertained from graphs of CSR versus N_1^{60}.

EXAM ORIENTATED SUMMARY FOR SARMA'S

(4) ASSESSMENT OF LIQUEFACTION POTENTIAL

① Cyclic stress ratio CSR

$$CSR = \tau/\sigma' = 0.65\, a_{max}(\gamma h/\sigma')\, r_d$$

$r_d = 1 - 0.00765h$

$\sigma' = (\gamma_{sat} - \gamma_w)h$

a_{max} = PGA from attenuation relationship

② Cyclic resistance ratio CRR

$$CRR = f(N_1^{60},\ M_w\ \text{fines content})$$

N_1^{60} = normalised SPT value

$= N C_N E/60$

C_N = depth normalisation factor

$= (100/\sigma')^{1/2}$

N = measured blow count

E = energy efficiency of the SPT (%)

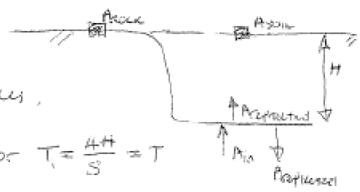
(5) SOIL LAYER RESPONSE

(a) Travelling wave technique

① $A_{in} = A_{ROCK}/2$

② $A_{REFRACTED} = \dfrac{2}{1+r} A_{in}$ and $A_{REFLECTED} = \dfrac{1-r}{1+r} A_{in}$; $r = \dfrac{S_s \rho_s}{S_R \rho_R}$

③ $A_{SOIL} = 2 A_{REFRACTED}\, e^{-\lambda\Omega z/S}$



Superposing multiple wave amplitudes,

$$A_s = A_{ROCK}\frac{2e^{-\lambda\pi/2}}{(1+r)\left[1 - \frac{1-r}{1+r}e^{-\lambda\pi}\right]} \quad \text{for } T_1 = \frac{4H}{S} = T$$

(b) SRSS technique

n	$\phi_n = \frac{4}{(2n-1)\pi}$	T	$\omega = 2\pi/T$	$\alpha = \frac{\Omega}{\omega}$	λ_{SOIL}	$\bar{\lambda}$	$\lambda_{TOTAL} = \lambda_{SOIL} + \bar{\lambda}$
1		$\frac{4H}{S} = T_1$	$2\pi/T_1$				
2		$T_1/3 = T_2$	$2\pi/T_2$				
3		$T_1/5 = T_3$	$2\pi/T_3$				
4		$T_1/7 = T_4$	$2\pi/T_4$				

Ω given i.e. forcing frequency or rock waves

$S_a \quad \phi^2 S_a^2$

$$A_{max} = \sqrt{\Sigma \phi^2 S_a^2} \times PGA_{ROCK}$$

$\Sigma\, \phi^2 S_a^2$

(c) Nonlinear behaviour (critical acceleration)

Tutorial: (SKS- MEng EE)

1. **Dynamic soil properties:**

Derive the formula for the undrained strength c_u in terms of the effective shear strength parameters (c',ϕ'), consolidation pressure p', coefficient of anisotropic consolidation K_0 and the cyclic pore pressure parameter A_n.

2. **Liqefaction hazard:**

$\gamma = 18$ kN/m^3
water table at surface for most critical

A civil engineering project site is selected close to an active fault at a distance of 25 km which is capable of producing earthquakes of maximum magnitude $M_w = 7$. Investigation of the site showed that there is a layer of loose sand at a depth of 6m. SPT tests gave a blow count of 15 in this soil. If the efficiency of the SPT tests is 50%, determine whether the site is likely to liquefy for the maximum earthquake. Is the site likely to liquefy with an earthquake of magnitude $M_w = 6.5$. What is the expected peak acceleration at the site ? (use the attenuation relationship $\log(A_g) = -1.09+0.238M_s-\log(R)-0.005R$ where $R^2= D^2+6^2$. You may assume that M_s is very close to M_w for the range of magnitude being considered).

3. **Soil layer response:**

A horizontal soil layer, 20m deep with a shear wave velocity of 100 m/sec is being subjected to an earthquake. The shear wave velocity in the rock below is 500 m/sec. If the free field surface acceleration in the rock is of simple harmonic type of period 0.8 sec and of amplitude 0.2g,

(a) If the soil is undamped, determine the acceleration on the soil surface in the first cycle. (answer=.33g) (Note- subsequent cycles can be determined only by plotting)

(b) If the soil is damped at 5% critical, what will be the amplitude in the first cycle. (answer-0.31g)

You may assume that the densities of the rock and the soil are the same.

(Note: Consider the travelling wave - impedance effect and then the refracted wave travelling into the soil. When the soil is damped- the amplitude decreases with distance travelled by the factor $\exp(-\lambda\Omega s/S)$ where λ= damping as a fraction of critical, Ω= frequency of the travelling wave, s= distance travelled and S = shear wave velocity in the soil.- Remember- the data is given as free field rock motion- be careful.)

4. Show that the amplitude of the surface motion of a damped soil layer in the steady state when subjected to simple harmonic motion of resonant frequency (resonance with the first mode of the soil layer) is A_s which is:

$$A_s = A_r \frac{2\,e^{-\lambda\pi/2}}{(1+r)\left[1-\frac{1-r}{1+r}e^{-\lambda\pi}\right]}$$

where A_r = free-field rock amplitude;
A_s = Amplitude of motion on the soil surface.

r = radiation coefficient between the soil and the rock= $S_s\rho_s/S_r\rho_r$
λ= Damping in the soil as fraction of critical.

Also, derive the result when the incoming wave is of the period resonating with the second mode of the layer.

(Hint: Plot the incoming & reflected waves vs. time to see a geometric series develop)

5. For the problem given in 3, determine the maximum acceleration in the steady state on the soil surface using the SRSS technique. Use four modes. Soil damping is 5% critical . The equivalent viscous damping for the radiation damping may be given as :

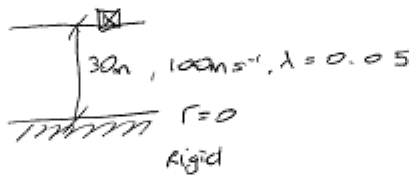
$$\overline{\lambda} = \frac{2r}{(2n-1)\pi}$$

where $\overline{\lambda}$ = Equivalent viscous damping as a fraction of critical (in addition to the damping in the soil itself).
n = mode number and r= radiation coefficient.
The acceleration spectrum of a harmonic acceleration in steady state is given as:

$$S_a = \frac{\sqrt{1+4\lambda^2\alpha^2}}{\sqrt{(1-\alpha^2)^2+4\lambda^2\alpha^2}}$$ and the mode shapes are $\Phi_n = 4/[(2n-1)\pi]$ on the surface.

where S_a = Spectral value as magnification of ground acceleration
α = Ω/ω
Ω = Forcing frequency
ω = frequency of the system and
λ = over all damping as a fraction of critical.
(Refer to dynamics of SDF systems to determine the acceleration spectrum of a harmonic acceleration input.) Answer= .73g

6.



$T_1 = 4H/V_s = 1.2 s$

$T_2 = T_1/3 = 0.4 s$

$T_3 = T_1/5 = 0.24 s$

$T_4 = T_1/7 = 0.17 s$

$x_{max} = \sqrt{\sum \phi^2(x,n)\, S_a^2(n)}$

2. $D = 25km$

$M_w = 7$

$z = 6m$

$N = 15$

$E = 50\%$

$\log(A_g) = -1.09 + 0.238 M_s - \log(R) - 0.005R$, $R^2 = 0^2 + 6^2$.

$N_1^{60} = N C_N \frac{E}{60}$

$= 15\sqrt{\frac{100}{\sigma'}}\ \frac{50}{60}$

$\sigma' = \gamma' h$

$= (\gamma_{SAT} - \gamma_w)h$

$= (18 - 9.8)(6)$

$= 49.2$

$\therefore N_1^{60} = 17.8$

$\log(A_g) = -1.09 + 0.238(7) - \log(\sqrt{25^2+6^2}) - 0.005\sqrt{25^2+6^2}$

$= -0.96365$

$A_g = 0.109$

≈ 0.11 ← no g

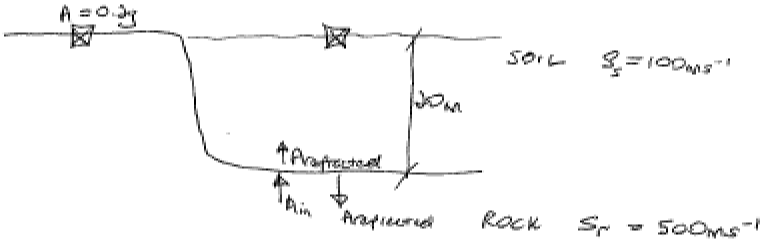
$\therefore CSR = 0.65\ a_{max}\ (\gamma h/\sigma')\ r_d$

$= 0.65\ (0.11)\ (18 \times 6 / 49.2)\ (1 - 0.00765 \times 6)$

$= 0.150$

From graph of CSR vs N_1^{60}, no liquefaction hazard.

3.



$A_{in} = A_{ROCK} / 2 = 0.2g / 2 = 0.1g$

note orientation ! Second medium over first medium

$$A_{refracted} = \frac{2}{1+r} A_{in} \quad , \quad r = \frac{S_s \rho_s}{S_r \rho_r} = \frac{100}{500} = 0.2$$

$$= \frac{2}{1.2}(0.1g)$$

$$= 0.167g$$

(a) No damping

$$A_{soil\ surface} = 2 A_{refracted}$$

$$= 0.33g .$$

(b) Damping

$$A_{soil\ surface} = 2A_{refracted}\, e^{-\lambda \Omega s/S}$$

$\lambda = 0.05$ note !

$$\Omega = 2\pi f$$

$$= \frac{2\pi}{T} = \frac{2\pi}{0.8}$$

$$= 7.85 \text{ rad/s} .$$

note

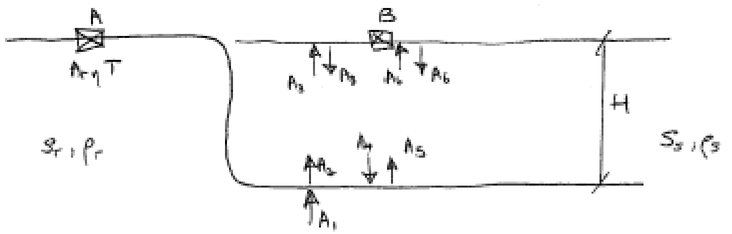
$s = 20$ m

$S_s = 100\ ms^{-1}$.

$$\therefore A_{soil\ surface} = 2\,(0.167g)\,(0.925)$$

$$= 0.31g .$$

4.



$$A_1 = A_r / 2 \qquad \Gamma = \frac{S_s \rho_s}{S_r \rho_r}$$

$$A_2 = \frac{2}{1+\Gamma} A_1 = \frac{A_r}{(1+\Gamma)} \qquad \bar{\Gamma} = \frac{S_r \rho_r}{S_s \rho_s} = \frac{1}{\Gamma}$$

$$A_3 = A_2 e^{-\lambda \Omega s/S}$$

$$= \frac{A_r}{(1+r)} e^{-\lambda \Omega s/S}$$

$$A_4 = A_3 e^{-\lambda \Omega s/S}$$

$$= \frac{A_r}{(1+r)} e^{-2\lambda \Omega s/S}$$

$$A_5 = \frac{1-\bar{r}}{1+\bar{r}} A_4$$

$$= \frac{1-\frac{1}{r}}{1+\frac{1}{r}} A_4$$

$$= \frac{r-1}{r+1} A_4$$

$$= \frac{r-1}{r+1} \frac{A_r}{(1+r)} e^{-2\lambda \Omega s/S}$$

$$A_6 = A_5 e^{-\lambda \Omega s/S}$$

$$= \frac{r-1}{r+1} \frac{A_r}{(1+r)} e^{-3\lambda \Omega s/S}$$

$$A_{A15} = 2A_3 = \frac{2A_r}{(1+r)} e^{-\lambda \Omega s/S}$$

note minus because phase change of 180° when soil to rock interface encountered.

$$A_{B15} = -2A_6 = -2\frac{r-1}{r+1} \frac{A_r}{(1+r)} e^{-3\lambda \Omega s/S}$$

$$\text{Geometric ratio} = \frac{A_{B15}}{A_{A15}} = -\frac{r-1}{r+1} e^{-2\lambda \Omega s/S} = \frac{1-r}{1+r} e^{-2\lambda \Omega s/S}$$

Geometric progression summation, $A_s = \dfrac{a}{1 - \text{geometric ratio}}$

$$= \frac{\dfrac{2A_r}{(1+r)} e^{-\lambda \Omega s/S}}{1 - \left(\dfrac{1-r}{1+r}\right) e^{-2\lambda \Omega s/S}}$$

$$= A_r \frac{2e^{-\lambda \Omega s/S}}{(1+r)\left[1 - \left(\dfrac{1-r}{1+r}\right) e^{-2\lambda \Omega s/S}\right]}$$

Note that when the soil layers are subjected to the resonant fundamental frequency,

$$\Omega = \frac{2\pi}{T} = \frac{2\pi}{\frac{4H}{S}} = \frac{2S\pi}{4H} = \frac{\pi S}{2H}$$

$$\therefore \quad A_s = A_r \frac{2e^{-\lambda \pi/2}}{(1+r)\left[1 - \dfrac{1-r}{1+r} e^{-\lambda \pi}\right]}$$

5.

$H = 20\,m$

$S_s = 100\,ms^{-1}$

$S_r = 500\,ms^{-1}$

$T_f = 0.8$

$A_r = 0.2g$

$\lambda_{soil} = 0.05$ (soil damping)

$$\bar{\lambda} = \frac{2r}{(2n-1)\pi}$$

$$r = \frac{S_s \rho_s}{S_r \rho_r} = \frac{100}{500} = 0.2$$

$$\Phi_n = \frac{4}{(2n-1)\pi}$$

forcing frequency

$$S_a = \frac{\sqrt{1+4\lambda^2\alpha^2}}{\sqrt{(1-\alpha^2)^2+4\lambda^2\alpha^2}} \quad ; \alpha = \frac{\Omega}{\omega} \quad , \Omega = \frac{2\pi}{0.8} = 7.8540\ rad/s$$

forcing period (given)

modal periods

n	ϕ_n	T (s)	ω (rad/s) $= 2\pi/T$	$\alpha = \frac{\Omega}{\omega}$	$\bar{\lambda}$
1	1.2732	$\frac{4H}{S} = 0.8000$	7.8540	1	0.1273
2	0.4244	$\frac{0.8}{3} = 0.2667$	23.5590	0.3334	0.0424
3	0.2546	$\frac{0.8}{5} = 0.1600$	39.2699	0.2000	0.0255
4	0.1819	$\frac{0.8}{7} = 0.1143$	54.9710	0.1429	0.01819

$\lambda_{TOTAL} = \lambda_{SOIL} + \bar{\lambda}$	S_a	$\phi_n^2 S_a^2$
0.17732	2.9917	14.5103
0.09244	1.124456	0.2277
0.07547	1.04163	0.0704
0.06819	1.02083	0.03447

$\Sigma = 14.8284$

$$S_a = \sqrt{\Sigma_i \phi^2 S_a^2} = \sqrt{14.84284} = 3.8526$$

$$\therefore A_{MAX_{SOIL}} = S_a\, PGA_{ROCK} = 3.8526 \times 0.2g = 0.7705g$$

1.1.3 Conceptual Structural Design for RC Structures in Seismic Regions [1]

1.1.3.1 Plan Layout

Rectangular plan shapes are preferable to winged, T, L or U shapes. Winged structures and structures with re-entrant corners suffer from non-uniform ductility demand distribution. Torsional effects are evident when the centre of mass (centre of application of inertial loads) and the centre of stiffness are offset. A couple which increases the shear forces on the columns then occur.

Also, extended buildings in plan are more susceptible to incoherent earthquake motion, being founded on different foundation material. Aspect ratio 1 to 3 at most, otherwise use seismic joints.

Building extended in plan will be subjected to asynchronous motion especially when founded on soft ground and should be avoided. Otherwise, seismic joints should be used to separate parts of the building along a vertical plane. In which case, the minimum separation between two adjacent parts should be calculated to ensure that no pounding occurs. Re-entrant corners resulting from T, L or U plan shapes attract high demands and should be avoided. The plan stiffness and strength distribution should be carefully checked and should be close to the centre of mass to avoid undue damaging effects of rotational response.

1.1.3.2 Elevation

The aspect ratio of the building in elevation affects the overturning moment exerted on the foundations. Low height to width aspect ratio is desirable.

Also, very slender structures suffer from higher mode contributions, thus necessitating the use of more elaborate seismic force calculations.

Differences of more than 20-25% in mass or stiffness between consecutive floors should be avoided. Mass concentrations should be avoided. Stiffness discontinuity (soft storeys) should be avoided i.e. higher storeys or a storey without infill panels whilst all other storey has should be avoided. Bridges between structures should be on rollers to minimize interaction. Irregularities in elevation exert concentrated ductility demand such as when a multi-bay tall building is taller in just one of the bays instead of all the bays.

The stiffness difference between two consecutive storeys should not exceed 25%. This not only implies that column sizes should not change drastically but also means that set-backs should be kept to a minimum. The total mass per floor should also conform to the same limits of variation. No planted columns should be allowed and no interruption of shear walls, where provided, should be permitted. Where column size and reinforcement is reduced with height, extreme care should be exercised at the location of such a change to ensure a uniform distribution of ductility demand. The aspect ratio in elevation should not be excessive; a height to width ratio of about 5:1 is reasonable. Whenever possible, the foundation level should be kept constant to avoid excessive demand being imposed on the shorter columns responding mainly in shear.

1.1.3.3 Beam and Column Axes

All beams and columns should have the same axes with no offset between adjacent members. Avoid columns supported on beams as imposed local demand in shear and torsion is considerable. Avoid partial infill panels as these create short columns susceptible to failure by shear cracking, just like link beams.

[1] ELNASHAI, *Earthquake Engineering Seismic Analysis Lectures.* Imperial College of Science Technology and Medicine, London, 2001.

1.1.3.4 Foundation Design

Forces on footings and bearing capacity underneath footings should be calculated from the worst case scenario of dead and live load combined with maximum over-turning moment and vertical earthquake component. For simplicity, the base shear may be applied at a point 2/3 of the height of the building from the base for the calculation of over-turning moment.

When there is no raft (mat) foundations, footings should be tied by RC beams with minimum reinforcement ratios. The point of connection of the tie beams should not be higher than the soffit of the footing, since this creates a short member which may be susceptible to shear failure.

1.1.3.5 Columns

Plastic hinges are allowed at the base of ground floor columns, but not at their tops. Transverse reinforcement should be provided in the lower ¼ of the column height at a minimum spacing. This may be increased in the middle region, but should be decreased again at the top of the column. Stirrups are all to have bending angle of 135 degrees or more, since 90 degrees stirrups are proven to be ineffective. Alternatively, the EC8 detailing requirements should be followed.

Strictly, no hinging columns above the ground floor should be allowed. In practice though, observations have indicated that limited hinging in a limited number of columns within a storey may be tolerated provided that a storey sway mechanism is completely avoided. The top and bottom ¼ column heights should be adequately confined by closed stirrups. These should be bent as above.

1.1.3.6 Member Capacity at Connections

To realize the weak beam and strong column response mode, beam capacity should be about 20%-25% lower than the capacity of the column, taking into account a conservative estimate of the effective width of the slab. This is one of the most important aspects of capacity design for seismic action. In this calculation, the actual areas of steel used should be accounted for, and not the design value. Also, this calculation should not include any material partial safety factors. Finally, for conservatism, the beam steel yield should be increased by about 10% to account for strain hardening. In one of each four columns in a storey, this condition may be relaxed. Also, this condition does not apply to top floor columns, where the rule can be relaxed.

The resistance mechanism in connections should be checked for the loading case with minimum gravity loads and maximum overturning, to ascertain that there is sufficient shear resistance even when the concrete contribution is at a minimum. The closely spaced stirrups used for column head and base should be continued into the beam-column connection. Also, beam reinforcement should be carefully anchored, especially in exterior connections. If the development length is sufficient, a column stub should be used to anchor the beam reinforcement.

1.1.3.7 Floor Slabs

Large openings should be avoided since the slabs are responsible for distributing the floor shear force amongst columns and reductions in their stiffness is not conducive to favourable seismic performance due to loss of diaphragm action.

1.1.3.8 Infill Panels

Masonry or block concrete infill panels should be protected against dislocation and shedding. Where provided, they should not be interrupted as to form short columns in adjacent members.

1.1.3.9 Building Separation

For adjacent buildings, a minimum separation is essential. This may be calculated according to a seismic code. A conservative approach is to calculate the elastic displacement under the design event and multiply it by the behaviour factor used in design. The resulting displacement should not lead to pounding at any level. If the structure in the vicinity is of drastically different dynamic characteristics, then its displacement, calculated as above, should be added to that of the building under consideration to arrive at the required gap width. For structures of largely similar characteristics founded on similar soils, some proportion of the sum of the two displacements may be used. There is no global agreement on this proportion.

1.1.3.10 Architectural Elements

These are part of the structure and should be treated as such. They should either be designed to resist the forces and especially the deformations imposed on them, or separated from the lateral load resisting system. The consequence of their damage and shedding should be verified with regard to life safety and interruption of use.

1.1.3.11 General Robustness

All parts of the structure should be tied to ensure a monolithic response under transverse vibrations. This includes all structural and non-structural components in the two orthogonal directions as well as the two directions one to the other.

1.1.3.12 Detailing Requirements and Ductile Response

There is a clear definition of the inter-relation between local detailing and local and global ductility of RC structures in EC8. For zones of low seismic exposure (say up to ground acceleration of 0.1g) ductility class L or M (low or medium) of EC8 may be used. Requirements for ductility class H (high) may be useful only in the case of structures where the designer requires a serious reduction in design forces, hence the use of an exceptionally high behaviour factor q.

1.1.3.13 Eurocode 8 Conceptual Design

B1 General

(1) The possible occurrence of earthquakes must be an important aspect to be accounted for in the conceptual design of a building in a seismic region.

(2) Such aspect has to be taken in consideration in the early stages of development of the building design, thus enabling the achievement of a structural system which , within acceptable costs, satisfies the fundamental requirements according to clause 2.1 of Part 1-1.

(3) To this end, the conceptual design of buildings in seismic regions should, as much as possible, reflect the considerations described in B2 - B7.

B2 Structural simplicity

(1) Structural simplicity, characterized by the existence of clear and direct paths for the transmission of the seismic forces, is an important objective to be pursued since the modelling, analysis, dimensioning, detailing and construction of simple structures are subject to much less uncertainty and thus the prediction of its seismic behaviour is much more reliable.

B3 Uniformity and symmetry

(1) Uniformity, which is somehow related to simplicity, is characterized by an even distribution of the structural elements which, when fulfilled in-plan, allows short and direct transmissions of the inertia forces created in the distributed masses of the building. If necessary, uniformity may be realized by subdividing the entire building by seismic joints into dynamically independent units.

(2) Uniformity in the development of the structure along the height of the building is also relevant, since it tends to eliminate the occurrence of sensitive zones where concentrations of stresses or large ductility demands might prematurely cause collapse.

(3) A close relationship between the distribution of masses and the distribution of resistance and stiffness naturally eliminates large eccentricities between mass and stiffness.

(4) In symmetrical or quasi-symmetrical building configurations, symmetrical structural layouts, well distributed in-plan, are thus obvious solutions for the achievement of uniformity.

(5) Finally, the use of evenly distributed structural elements increases redundancy and allows for a more favourable redistribution of action effects and widespread energy dissipation across the entire structure.

B4 Bidirectional resistance and stiffness

(1) Horizontal seismic motion is a bidirectional phenomenon and thus the building structure must be able to resist horizontal actions in any direction. Accordingly, the structural elements should be arranged in such a way as to provide such resistance which, usually, is achieved by organizing them within an orthogonal in-plan structural mesh and ensuring similar resistance and stiffness characteristics in both main directions.

(2) Furthermore, the choice of the stiffness characteristics of the structure, while attempting to minimize the effects of the seismic action (taking into account its specific features at the site) should also limit the development of excessive displacements that might lead to instabilities due to second order effects or large damages.

B5 Torsional resistance and stiffness

(1) Besides lateral resistance and stiffness, building structures must possess adequate torsional resistance and stiffness in order to limit the development of torsional motions which tend to stress, in a non-uniform way, the different structural elements. In this respect, arrangements in which the main resisting elements are distributed close to the periphery of the building present clear advantages.

B6 Diaphragm action at storey level

(1) In buildings, floors play a very important role in the overall seismic behaviour of the structure. In fact, they act as horizontal diaphragms that, not only collect and transmit the inertia forces to the vertical structural systems but also ensure that those systems act together in resisting the horizontal action.

(2) Consequently, floors are an essential part of the whole building structure and naturally its diaphragm action is especially relevant in cases of complex and non-uniform layouts of the vertical structural systems or when systems with different horizontal deformability characteristics are used together (e.g. dual systems).

(3) It is thus of the utmost importance that the floor systems be provided with adequate in-plan stiffness and resistance and with efficient connections to the vertical structural systems. In this respect, particular care should be taken in the cases of non-compact or very elongated in-plan shapes and in the case of existence of large floor openings, especially if the latter are located in the vicinity of the main vertical structural elements thus hindering such efficient connection.

B7 Adequate foundation

(1) With regard to the seismic action the design and construction of the foundations and of the connection to the superstructure shall ensure that the whole building is excited in a uniform way by the seismic motion.

(2) Thus, for structures composed of a discrete number of structural walls, likely to differ in width and stiffness, a rigid, box-type or cellular foundation, containing a foundation slab and a cover slab, should be chosen.

(3) For buildings with individual foundation elements (footings or piles) the use of a foundation slab or tie-beams between these elements in both main directions should be considered, subject to the criteria of clause 5.4.1.2 of Part 5.

1.1.4 Methods of Structural Analysis

Earthquake excitations are random non-stationary functions starting from a low-level building up to a maximum then dying away. Exact solution methods are not established. Instead, we could either analyze a set of such events using either

I. The **Equivalent Lateral Force Response Spectrum** method described in **Section 1.1.5** (fundamental mode, linear, elastic analysis). This method automatically performs the seismic hazard assessment (of **Section 1.1.1.18**) and evaluates the effect of the soil layers (of **Section 1.1.2.15.1**) culminating in the response spectrum before the structural analysis is carried out.

II. The **Multi-Modal Response Spectrum** method described in **Section 1.1.5** (higher modes, linear, elastic analysis). This method automatically performs the seismic hazard assessment (of **Section 1.1.1.18**) and evaluates the effect of the soil layers (of **Section 1.1.2.15.1**) culminating in the response spectrum before the structural analysis is carried out.

III. **Performance Based Seismic Analysis and Design**

  (a) **Multi-Modal Response Spectrum (With No Behaviour Factor)** methodology described in **Section 1.1.5** and **Section 1.1.8**.

  (b) **Random (Performance Based Seismic Engineering)** solution method assuming a Gaussian and stationary (and ergodic) excitations as described in **Section 1.1.8**. This method requires the derivation of the input power spectrum (appropriately scaled to match the soil elastic response spectra) for the earthquake at soil level (from the seismic hazard assessment of **Section 1.1.1.18** and effect of the soil layers of **Section 1.1.2.15.1**) to be applied onto the structure model or alternatively the input power spectrum (appropriately scaled to match the rock elastic response spectra) for the earthquake at rock level (from the seismic hazard assessment of **Section 1.1.1.18**) to be applied onto the soil and structure model. The latter soil and structure model clearly accounts for the soil-structure interaction (kinematic and inertial interaction) effect.

  (c) **Deterministic Transient Solution (Performance Based Seismic Engineering)** methods based on appropriately scaled (to match elastic response spectra) earthquake time histories and performing linear (as described in **Section 1.1.8**) or nonlinear (as described in **Section 1.1.8**) transient dynamic analyses and then enveloping the results (higher modes, nonlinear, inelastic analysis). This method requires the derivation of the input time histories (appropriately scaled to match the soil elastic response spectra) for the earthquake at soil level (from the seismic hazard assessment of **Section 1.1.1.18** and effect of the soil layers of **Section 1.1.2.15.1**) to be applied onto the structure model or alternatively the input time histories (appropriately scaled to match the rock elastic response spectra) for the earthquake at rock level (from the seismic hazard assessment of **Section 1.1.1.18**) to be applied onto the soil and structure model. The latter soil and structure model clearly accounts for the soil-structure interaction (kinematic and inertial interaction) effect.

1.1.5 GL, ML Shock and Response Spectrum Analysis

1.1.5.1 Nature of the Dynamic Loading Function

The solution method can be used to solve dynamic systems subjected to: -

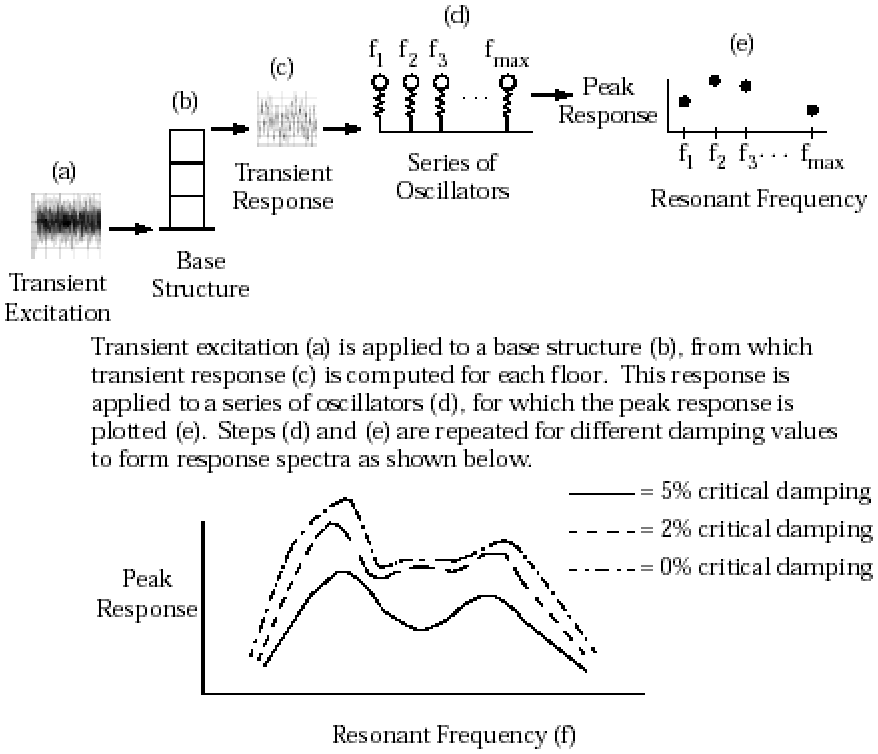
(a) **Random non-stationary short duration impulse** loading functions

Deterministic loadings can be readily solved in the frequency or time domain using forced frequency and forced transient response analyses. Random loadings, which are stationary and ergodic, can be solved in the frequency domain using random vibration analysis. If the forcing function is a random non-stationary forcing function such that the random forces start from a low-level building up to a maximum then dying away, such as in a seismic event, then exact solution methods are not established. Instead, we could either analyze a set of such events using deterministic transient solution methods and then average or envelope the results or alternatively use the response or shock spectrum method which envelopes the response spectra of a series of time histories. The latter method, although computationally cheaper, ignores phase information of the signals. Response spectrum computes the relative response with respect to the base whilst shock spectrum computes the absolute response.

In this LINEAR TIME DOMAIN solution, the static response must be added to the dynamic response if the dynamic analysis is performed about the initial undeflected (by the static loads) state with only the dynamic loads applied, hence causing the dynamic response to be measured relative to the static equilibrium position. **Hence, the total response = the dynamic response + the static response to static loads.**

1.1.5.2 The Response Spectra

Response spectrum analysis is an approximate method of computing the peak response of a transient excitation applied to a simple structure or component. If is often used for earthquake excitations and also to predict peak response of equipment in spacecraft that is subjected to impulsive load due to stage separation.



Transient excitation (a) is applied to a base structure (b), from which transient response (c) is computed for each floor. This response is applied to a series of oscillators (d), for which the peak response is plotted (e). Steps (d) and (e) are repeated for different damping values to form response spectra as shown below.

A response spectrum is a curve of the maximum response (displacement, velocity, acceleration etc.) of a series of single DOF systems of different natural frequencies and damping to a given acceleration time history. It characterizes the acceleration time history and has nothing to do with the properties of the structure. Any acceleration time series can be converted into a response spectrum. The response spectrum at rock level is obviously different from that on the soil surface because the acceleration time series would have been modified by the flexibility of the soil. Hence, the response spectrum is generated at the level where the structure stands. However, in doing so it becomes apparent that the characteristics of the soil no longer play any part in the dynamic response of the structure, i.e. there is no soil-structure interaction.

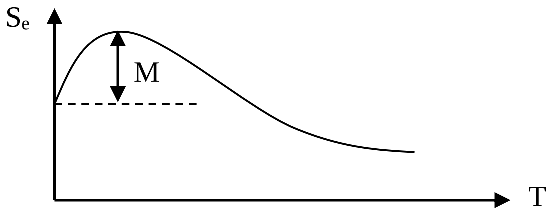
The response spectrum gives the maximum response for a series of SDOF systems characterized by their natural frequencies and damping. To establish a response spectrum, the dynamic equations of motion are solved for each and every SDOF system using Duhamel's integral for linear systems. Step-by-step linear acceleration method (explicit scheme since there is only one DOF) for nonlinear systems cannot really be employed, as superposition of nonlinear modal responses is not strictly valid for nonlinear systems where the physical coordinates (which includes all modes together) responds nonlinearly. One equation is solved for each SDOF system and the maximum value plots one point on the response spectra.

When analyzing the dynamic response of a structure founded on rock, the input motion due to an earthquake is the same with or without the structure. Calculations assuming a fixed base structure should therefore give realistic results. However, when analyzing a structure founded on a soil site due to the same earthquake there are changes in motion at foundation level leading to changes in dynamic response of the structure. These changes are all effects of dynamic soil structure interaction. The structure will interact with the soil in two ways. Firstly, structural inertial loads are transferred back into the soil and secondly, the stiffer structural foundations are not able to conform to the generally non-uniform motion of the free-field surface. These effects are known respectively as **inertial** and **kinematic interaction**. To account for soil-structure interaction, other methods such as random solutions or deterministic transient solutions with both the soil and structure explicitly modeled must be employed.

Once the response spectrum has been established, the response of a structure can be established depending on whether the idealization is SDOF (equivalent lateral force method) or MDOF (multi-modal spectrum analysis).

1.1.5.2.1 Ball Park Figures

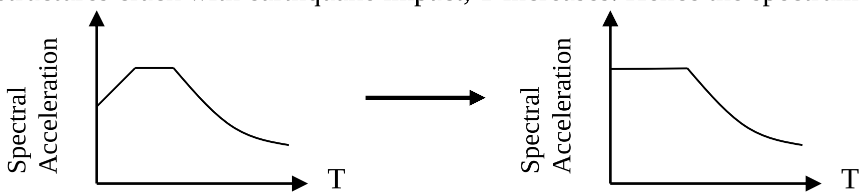
Wind loading base shear, $V_{wind} = 0.01 – 0.03\ W_{TOTAL}$
Earthquake elastic base shear, $V_{elastic} = S_e(g)(T_1)\ W_{TOTAL} = 0.25 – 0.30\ W_{TOTAL}$ in high seismicity areas
Earthquake inelastic base shear, $V_{inelastic} = S_d(g)(T_1)\ W_{TOTAL} = 0.15 – 0.20\ W_{TOTAL}$ in high seismicity areas
$= 0.05 – 0.07\ W_{TOTAL}$ in low seismicity areas
Peak ground acceleration, PGA = 0.40g in high seismicity areas
= 0.07g in low seismicity areas
Spectrum amplification, M = 2.5



Behaviour factor, q = 2 for very non-ductile structures
= 8.5 for very ductile structures

1.1.5.2.2 Low Period Structures

As RC structures crack with earthquake impact, T increases. Hence the spectrum is modified as follows.

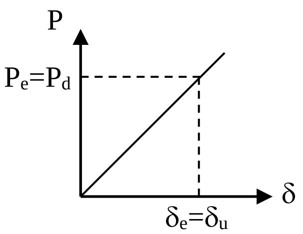


1.1.5.2.3 Behaviour Factors (Force Reduction Factors) for Inelastic Design

It is not feasible, practical or economical to design elastically. The accelerations obtained from elastic loading $P_e = S_e(g)(T)W$ could cause great nonstructural damage and endanger lives as $S_e(T) >> S_d(T)$. The concept is to use the ductility of the structure to absorb energy by designing for $P_d < P_e$. Concepts of energy absorption in the inelastic range are used to reduce the elastic forces by as much as 80 – 85%. Hence the force reduction factor (aka response modification factor or behaviour factor) is

$$\text{Behaviour factor, } q = \frac{S_e}{S_d}$$

The relationships between elastic and inelastic forces for different periods are



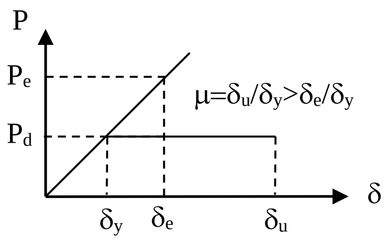
Equal Acceleration

Very Short Period

$T < 0.2s$

$P_d = P_e$

$q = 1.0$



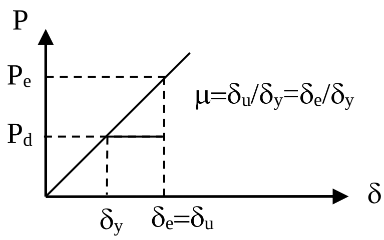
Equal Energy

Short Period

$0.2s < T < 0.5s$

$$P_d = \frac{P_e}{\sqrt{2\mu - 1}}$$

$$q = \sqrt{2\mu - 1}$$



Equal Displacement

Long Period

$T > 0.5s$

$$P_d = \frac{P_e}{\mu}$$

$q = \mu$

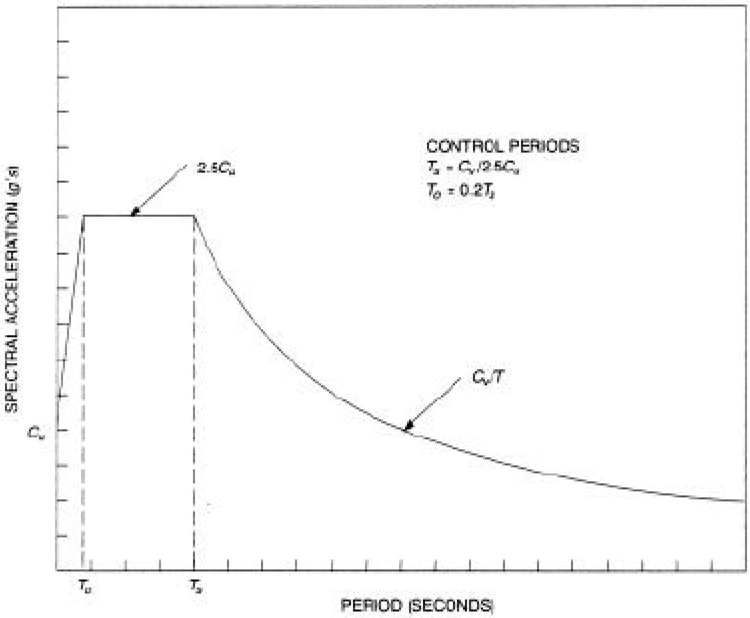
Hence, the behaviour factor depends on the period of the structure and to a lesser extent the period of maximum amplification in the earthquake spectrum. The force reduction factor q increases as T increases. The aspects that contribute to the value of q are

I. Ductility i.e. inelastic energy absorption through damage, which is equivalent to having higher damping (accounted for)
II. The ability of the structure to redistribute the action effects upon softening of a particular zone (accounted for)
III. Loss of energy through radiation damping in the soil semi infinite half space (not accounted for)
IV. Effect of uplift on cutting off the input of earthquake energy (not accounted for)

The response modification (reduction) behaviour factor q is generally on the conservative side for the force calculations, but may be on the unconservative side for displacements. Hence, for the **assessment of displacement**, the linear elastic displacements under the design forces **are multiplied (increased) by q** to arrive to an approximate inelastic displacement.

1.1.5.2.4 UBC 1997 Elastic and Inelastic Response Spectrum

The UBC elastic spectrum is defined by three lines governing low period response, mid period response and high period response.



Depending on where the mode period T lands the base spectral acceleration $S_a(g)(T)$ is defined by

$$T < T_0 \qquad a_{spectral} = C_a\left(1 + 1.5\frac{T}{T_0}\right)$$

$$T_0 < T < T_1 \quad a_{spectral} = 2.5C_a$$

$$T > T_1 \qquad a_{spectral} = \frac{C_v}{T}$$

where T_0 and T_1 depend on the soil.

$$T_1 = \frac{C_v}{2.5C_a}, \quad T_0 = 0.2T_1$$

The seismic coefficients C_a and C_v are dependent upon seismicity level and soil conditions at the site as follows.

Soil Profile Type	$Z = 0.075$		$Z = 0.15$		$Z = 0.20$		$Z = 0.30$		$Z = 0.40$	
	C_a	C_v	C_a	C_v	C_a	C_v	C_a	C_v	C_a	C_v
S_A	0.06	0.06	0.12	0.12	0.16	0.16	0.24	0.24	$0.32N_a$	$0.32N_v$
S_B	0.08	0.08	0.15	0.15	0.20	0.20	0.30	0.30	$0.40N_a$	$0.40N_v$
S_C	0.09	0.13	0.18	0.25	0.24	0.32	0.33	0.45	$0.40N_a$	$0.56N_v$
S_D	0.12	0.18	0.22	0.32	0.28	0.40	0.36	0.54	$0.44N_a$	$0.64N_v$
S_E	0.19	0.26	0.30	0.50	0.34	0.64	0.36	0.84	$0.36N_a$	$0.96N_v$
S_F *										

*Requires site-specific investigation and analysis to determine seismic coefficients

N_a and N_u are near-source factors for Seismic Zone 4. Their values are in the range of 1.0 to 2.0 as follows.

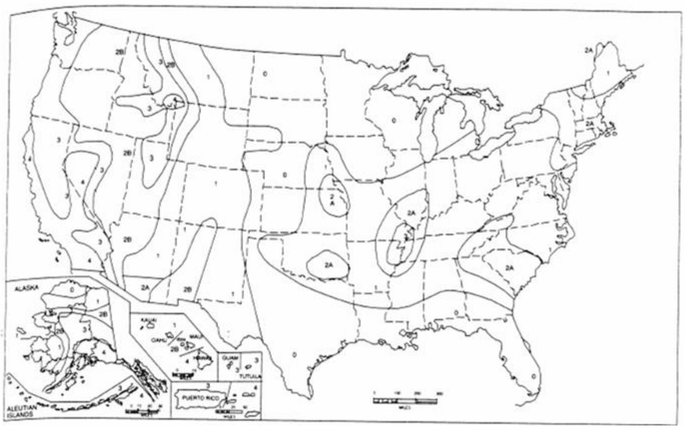
Near source factor N_a

Seismic Source Type	Closest distance to known seismic source		
	≤ 2km	5 km	≥ 10km
A	1.5	1.2	1.0
B	1.3	1.0	1.0
C	1.0	1.0	1.0

Near source factor N_v

Seismic Source Type	Closest distance to known seismic source			
	≤ 2km	5km	10km	≥ 15km
A	2.0	1.6	1.2	1.0
B	1.6	1.2	1.0	1.0
C	1.0	1.0	1.0	1.0

The zone factor Z represents the effective peak ground acceleration expected in the region and is in units of g. These values are obtained from seismic hazard analysis (as according to **Section 1.1.1.18**) culminating in the seismic zone map. The values reflect a 90% probability of not being exceeded in 50 years, which corresponds to a return period of 475 years. In the UBC, the US is divided into 5 seismic zones, namely, zones 1, 2A, 2B, 3, and 4.



Zone	1	2A	2B	3	4
Z	0.075	0.15	0.20	0.30	0.40

To account for the amplification effects introduced by different local soil conditions, the UBC distinguishes six different soil profile types as summarized as follows.

Soil Type	S_A	S_B	S_C	S_D	S_E	S_F *
Description	Hard Rock	Rock	Very Dense Soil and Soft Rock	Stiff Soil Profile	Soft Soil Profile	Soil Requiring Site-Specific Evaluation

*e.g., soils vulnerable to liquefaction

In the absence of detailed information, one may assume type S_D. Type S_E or S_F need not be considered unless it is substantiated by geotechnical investigations.

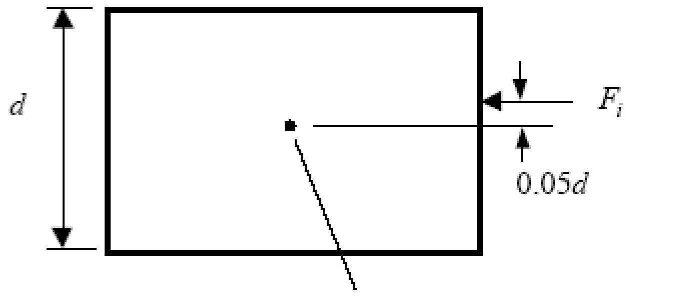
The elastic spectrum can be converted into the inelastic spectrum by dividing by the ductility response modification factor, R. The structural factor R reflects the global ductility of the lateral load resistance system of the structure. The more ductile the structural system is, the lower can be the resistance of the structure and therefore, the larger can be the value of R.

Structural System	*R*
Bearing Wall Systems with Concrete or Masonry Shear Walls	4.5
Building frame Systems with Concrete or Masonry Shear Walls	5.5
Special Steel or Concrete Moment-resisting Frame Systems	8.5
Ordinary Steel Moment-resisting Frame Systems	4.5
Ordinary Concrete Moment-resisting Frame Systems	3.5

Criteria for Design

The 1.4 x inelastic response spectrum is used for ultimate state plastic capacity design.

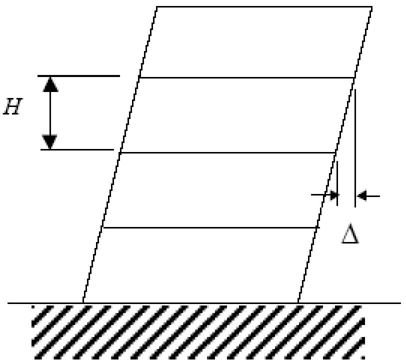
To account for **accidental torsional loads**, where floor diagrams are not flexible, the UBC requires that the force F_i at each level be applied at a point that is away from the calculated center of mass by a distance that is 5% of the building dimension at that level perpendicular to the direction of F_i.



$$M_x = k\,dF_x$$

where k = 5%.

The UBC limits the **story drift** in a structure. The story drift, Δ is the relative displacement between two adjacent levels of a structure.



The drift Δ_S is determined with the inelastic response spectrum. The maximum inelastic drift Δ_M is then computed

$$\Delta_M = 0.7R\Delta_S$$

For structures with T < 0.7 seconds,

$$\Delta_M \leq 0.025H$$

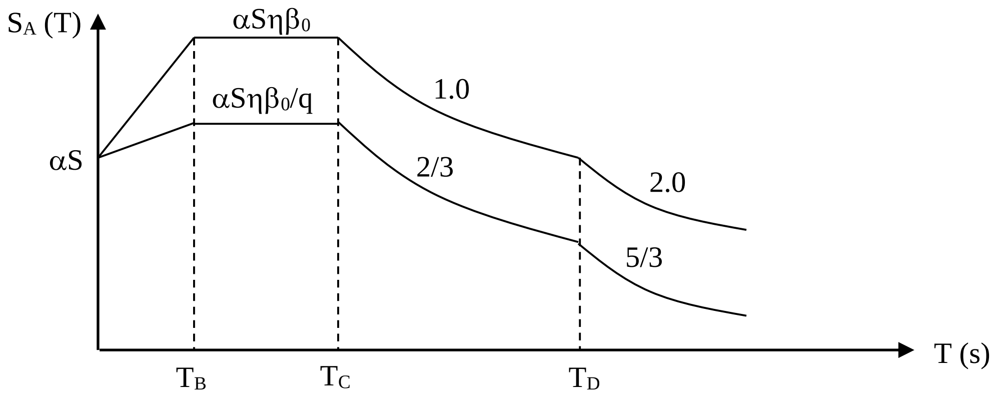
For structures with T > 0.7 seconds,

$$\Delta_M \leq 0.020H$$

where H is the storey height.

1.1.5.2.5 EC8 Elastic and Inelastic Response Spectrum

The elastic $S_e(g)(T)$ and inelastic spectrum $S_d(g)(T)$ are defined in four parts as follows. The response spectrum is defined for 10% probability of occurrence in 50 years, or a return period of 475 years. The following depicts the **horizontal** response spectrum. Note that these response spectra are **normalized by g**.



$$S_e(T) = a_g \cdot S \cdot \left[1 + \frac{T}{T_B} \cdot (\eta \cdot \beta_0 - 1)\right] \qquad S_d = \alpha S\left[1 + \frac{T}{T_B}\left(\eta\frac{\beta_0}{q} - 1\right)\right] \quad \text{for} \quad 0 < T < T_B$$

$$S_e(T) = a_g \cdot S \cdot \eta \cdot \beta_0 \qquad S_d = \alpha S \eta \frac{\beta_0}{q} \quad \text{for} \quad T_B < T < T_C$$

$$S_e(T) = a_g \cdot S \cdot \eta \cdot \beta_0 \cdot \left[\frac{T_C}{T}\right]^{k_1} \qquad S_d \begin{cases} = \alpha S \eta \frac{\beta_0}{q}\left[\frac{T_C}{T}\right]^{k_{d1}} \\ \geq 0.2\alpha \end{cases} \quad \text{for} \quad T_C < T < T_D$$

$$S_e(T) = a_g \cdot S \cdot \eta \cdot \beta_0 \cdot \left[\frac{T_C}{T_D}\right]^{k_1} \cdot \left[\frac{T_D}{T}\right]^{k_2} \qquad S_d \begin{cases} = \alpha S \eta \frac{\beta_0}{q}\left[\frac{T_C}{T_D}\right]^{k_{d1}}\left[\frac{T_D}{T}\right]^{k_{d2}} \\ \geq 0.2\alpha \end{cases} \quad \text{for} \quad T_D < T$$

where the damping correction factor η with reference value $\eta = 1$ for 5% viscous damping is

$$\eta = \sqrt{7/(2+\xi)} \geq 0.7$$

Note that α (or a_g) is the peak ground acceleration in g, S is the soil response parameter, β_0 is the spectral amplification factor for 5% damping, T_B and T_C are the limits of the constant acceleration region, T_D is the limit of the constant displacement region and k_1 and k_2 are the spectrum shape exponents.

The **vertical** response spectrum is obtained by scaling the horizontal response spectrum as follows.

— For vibration periods T smaller than 0,15 s the ordinates are multiplied by a factor of [0,70].

— For vibration periods T greater than 0,50 s the ordinates are multiplied by a factor of [0,50].

— For vibration periods T between 0,15 s and 0,50 s a linear interpolation shall be used.

Note that the **peak ground acceleration α (or a_g)** is the **acceleration at bedrock** level. This will be modified by the soil as depicted by the fact that at T = 0, the soil ground acceleration is αS. The soil would also affect the response spectrum for other values of T. The codified response spectra then represent the excitation signal at the soil level, and not at the bedrock level. A public domain database of the peak ground acceleration worldwide is maintained at GSHAP (http://www.seismo.ethz.ch/GSHAP/global/).

The soil class determined parameters are as follows.

Elastic response spectrum parameters

Subsoil Class	S	β_0	T_B [s]	T_C [s]	T_D [s]
A	1.0	2.5	0.10	0.40	3.0
B	1.0	2.5	0.15	0.60	3.0
C	0.9	2.5	0.20	0.80	3.0

Design response spectrum parameters

Subsoil Class	k_{d1}	k_{d2}
A	$^2/_3$	$^5/_3$
B	$^2/_3$	$^5/_3$
C	$^2/_3$	$^5/_3$

Corresponding values of k_1 and k_2 for the elastic spectra are 1.0 and 2.0 for all subsoil classes. The subsoil classes are defined as follows.

— **Subsoil class A**

— Rock or other geological formation characterized by a shear wave velocity v_s of at least 800 m/s, including at most 5 m of weaker material at the surface.

— Stiff deposits of sand, gravel or overconsolidated clay, at least several tens of m thick, characterized by a gradual increase of the mechanical properties with depth and by v_s-values of at least 400 m/s at a depth of 10 m).

— **Subsoil class B**

Deep deposits of medium dense sand, gravel or medium stiff clays with thickness from several tens to many hundreds of m, characterized by v_s-values of at least 200 m/s at a depth of 10 m, increasing to at least 350 m/s at a depth of 50 m.

— **Subsoil class C**

— Loose cohesionless soil deposits with or without some soft cohesive layers, characterized by v_s-values below 200 m/s in the uppermost 20 m.

— Deposits with predominant soft-to-medium stiff cohesive soils, characterized by v_s-values below 200 m/s in the uppermost 20 m.

When the subsoil profile includes an alluvial surface layer with thickness varying between 5 and 20 m, underlain by much stiffer materials of class A, the spectrum shape for subsoil class B can be used together with an increased soil parameter S equal to 1.4, unless a special study is performed.

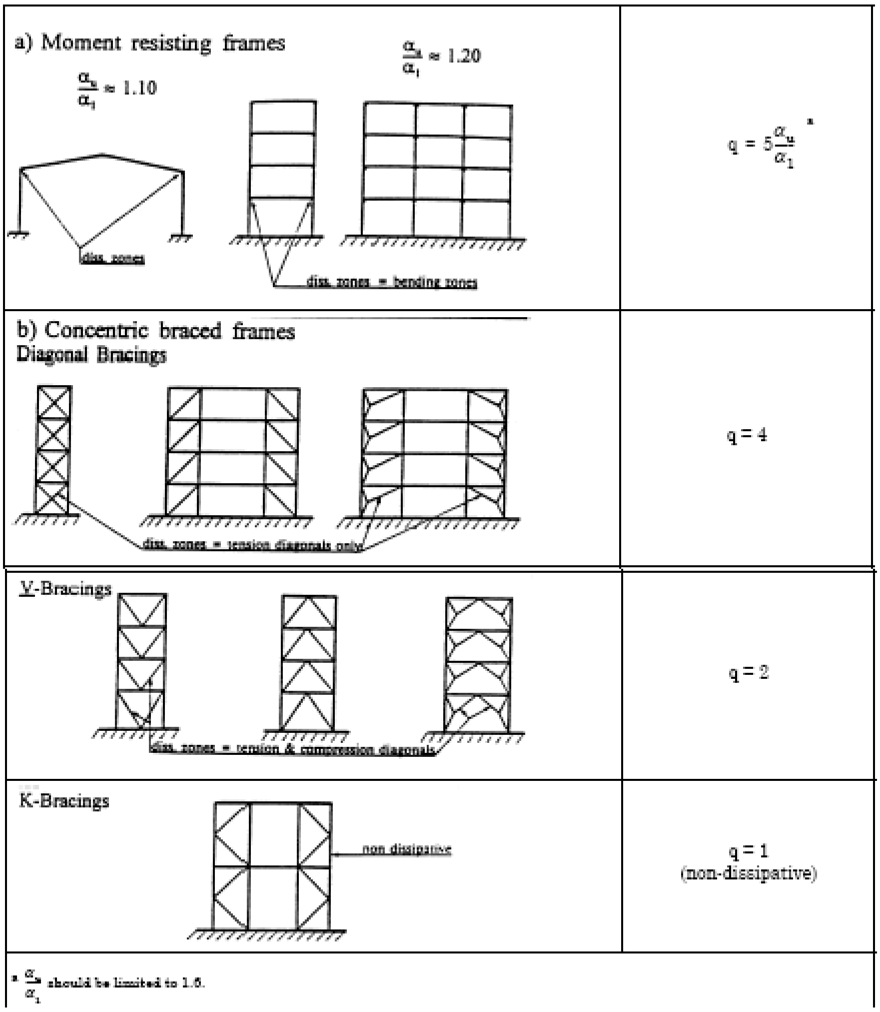
Special attention should be paid in the case of a deposit of subsoil class C which consists — or contains a layer at least 10 m thick — of soft clays/silts with high plasticity index (PI > 40) and high water content. Such soils typically have very low values of vs, low internal damping and an abnormally extended range of linear behaviour and can therefore produce anomalous seismic site amplification and soil-structure interaction effects; In this case, a special study for the definition of the seismic action should be carried out, in order to establish the dependence of the response spectrum on the thickness and vs-value of the soft clay/silt layer and on the stiffness contrast between this layer and the underlying materials.

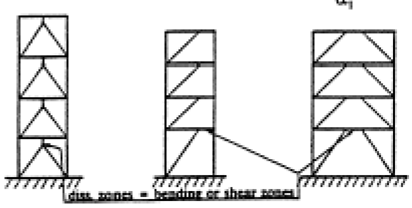
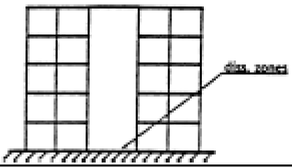
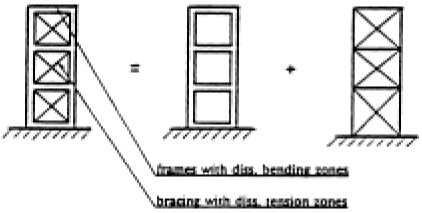
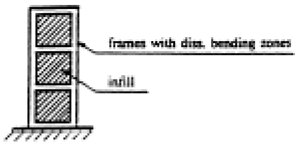
Typically, the behaviour factor q is unity for elastic response and may be up to 5 for RC structures and 6 for steel structures. This may be contrasted with values of R_w but after dividing by 1.4 (i.e. R_w maximum is 12/1.4 = 8.5 and minimum 4/1.4 = 2.8) in recognition of the different philosophies consistent with the behaviour factor used. Note that when q is greater than about 2.5, the inelastic spectrum dips below the zero period value (ground acceleration). Of course, the greater the value of q, the lower the base shear.

For **concrete** structures, $q = q_0 k_D k_R k_W$.

STRUCTURAL TYPE		q_o
Frame system		5,0
Dual system	frame equivalent	5,0
	wall equivalent, with coupled walls	5,0
	wall equivalent, with uncoupled walls	4,5
Wall system	with coupled walls	5,0
	with uncoupled walls	4,0
Core system		3,5
Inverted pendulum system		2,0

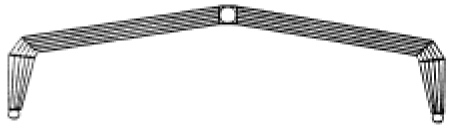
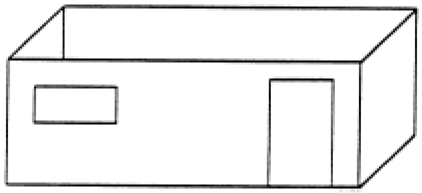
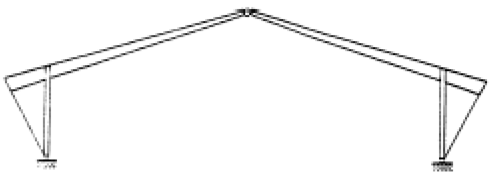
For **steel** structures q is given by the follows.

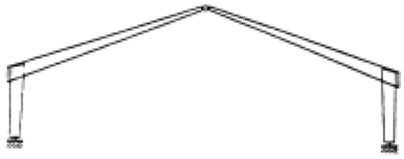
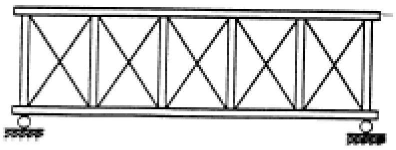
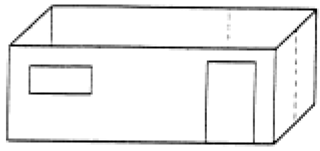
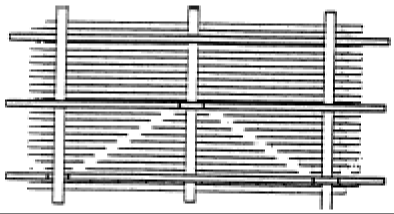
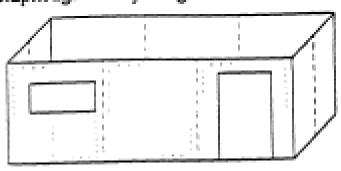


<table>
<tr><td>c) Eccentric braced frames $\frac{\alpha_u}{\alpha_1} \approx 1.10$

</td><td>$q = 5\frac{\alpha_u}{\alpha_1}$ *</td></tr>
<tr><td>d) Cantilever structures
Restrictions: $\bar{\lambda} \leq 1.5$; $\theta \leq 0.2$ (see clause 3.5.7)</td><td>q = 2</td></tr>
<tr><td>e) Structures with concrete cores or concrete walls

</td><td>see section 2</td></tr>
<tr><td>f) Dual structures

</td><td>$q = 5\frac{\alpha_u}{\alpha_1}$ *</td></tr>
<tr><td>g) Mixed structures (Steel moment resisting frames with reinforced concrete or masonry infills)

</td><td>q = 2</td></tr>
<tr><td colspan="2">* $\frac{\alpha_u}{\alpha_1}$ should be limited to 1.6.</td></tr>
</table>

For **timber** structures, the behaviour factor is listed

Type	Description	Behaviour Factor q
A	Non-dissipative structures	1,0
B	Structures having low capacity to dissipate energy	1,5
C	Structures having medium capacity to dissipate energy	2 0
D	Structures having good capacity to dissipate energy	3,0

Type	Description	q	Examples
A	**Non-dissipative structures** having only few mechanical fasteners beyond the dissipative zones	1,0	Arches with hinged joints   Cantilever structures with rigid connections at the base  Buildings with vertical diaphragms resisting the horizontal forces without mechanical fasteners for both interconnection and between sheathing and timber framing. 
B	**Structures having low capacity of energy-dissipation**	1,5	Structures with semi-rigidly fixed-based columns  Structures with few but effective dissipative zones 

Type	Description	q	Examples
C	**Structures having medium capacity of energy-dissipation**	2,0	Frames or beam-column structures with semirigid joints between all members. Connections with foundations may be semirigid or hinged (according to the load-carrying system)  Braced frame structures with mechanical fasteners in the joints of the frame and/or the connections of the bracing elements  Buildings with vertical diaphragms resisting the horizontal forces, where sheathing is glued to the framing. Diaphragms are interconnected by mechanical fasteners (horizontal diaphragms may be glued or nailed)  Mixed structures consisting of timber framing (resisting the horizontal forces) and non-load-bearing infillment 
D	**Structures having good capacity of energy-dissipation**	3,0	Buildings with vertical diaphragms resisting the horizontal forces, where sheathing is fixed to the framing by mechanical fasteners as well as the interconnection of the wall-elements (horizontal diaphragms may be glued or nailed) 

For **masonry** structures, the behaviour factor is listed.

Table 5.1 — Types of construction and behaviour factors

Type of construction	Behaviour factor q
Unreinforced masonry	[1,5]
Confined masonry	[2,0]
Reinforced masonry	[2,5]

For irregular buildings (in whatever material) q is further reduced by 20% from that listed above.

The only difference between the elastic and the inelastic spectrum is the behaviour factor q and the values of k1 and k2. To be more precise, the spectral amplification β_0 is divided by q in the inelastic spectrum.

EC8 requires the dynamic mass calculated (i.e. divide the gravitational force by g to get mass) for the determination of the periods of the structure and the mass being acceleration by the ground motion (the greater the mass, the greater the base shear) from

100% DL + $\varphi\psi_{2i}$ LL

where φ is obtained from

Table 3.2: Values of φ for calculating ψ_{Ei}

Type of variable action	Occupation of storeys		φ
Categories A-C*	storeys independently occupied	top storey other storeys	[1,0] [0,5]
Categories A-C*	some storeys having correlated occupancies	top storey storeys with correlated occupancies other storeys	[1,0] [0,8] [0,5]
Categories D-F* Archives			[1,0]

* Categories as defined in Part 1 of Eurocode 1

The peak ground displacement may be estimated from

$$d_g = [0,05] \cdot a_g \cdot S \cdot T_C \cdot T_D$$

The structural displacements can be determined from

$$d_s = q_d \cdot d_e \cdot \gamma_I$$

where q is the behaviour factor, d_e the displacement response and γ the importance factor.

The importance factors are presented.

Table 3.3: Importance categories and importance factors for buildings

Importance category	Buildings	Importance factor γ_I
I	Buildings whose integrity during earthquakes is of vital importance for civil protection, e.g. hospitals, fire stations power plants, etc.	[1,4]
II	Buildings whose seismic resistance is of importance in view of the consequences associated with a collapse, e.g. schools, assembly halls, cult. institutions etc.	[1,2]
III	Ordinary buildings, not belonging to the other categories	[1,0]
IV	Buildings of minor importance for public safety, e.g. agricultural buildings, etc.	[0,8]

Criteria for Design

Seismic zones with a design ground acceleration α (i.e. peak ground acceleration in g) not greater than 0.10g are low seismicity zones, for which reduced or simplified seismic design procedures for certain types or categories of structures may be used. In seismic zones with a design ground acceleration α not greater than 0.04g the provisions of Eurocode 8 need not be observed.

EC8 seismic load combinations cases are

$$\mathbf{1.0\ DL + \psi_{2i}\ LL \pm \gamma\ 1.0\ EQ_X}$$
$$\mathbf{1.0\ DL + \psi_{2i}\ LL \pm \gamma\ 1.0\ EQ_Y}$$
$$\mathbf{1.0\ DL + \psi_{2i}\ LL \pm \gamma\ 1.0\ (EQ_X^2 + EQ_Y^2)^{1/2}}$$
$$\mathbf{1.0\ DL + \psi_{2i}\ LL \pm \gamma\ 1.0\ EQ_X \pm \gamma\ 0.3\ EQ_Y \pm \gamma\ 0.3\ EQ_Z}$$
$$\mathbf{1.0\ DL + \psi_{2i}\ LL \pm \gamma\ 0.3\ EQ_X \pm \gamma\ 1.0\ EQ_Y \pm \gamma\ 0.3\ EQ_Z}$$
$$\mathbf{1.0\ DL + \psi_{2i}\ LL \pm \gamma\ 0.3\ EQ_X \pm \gamma\ 0.3\ EQ_Y \pm \gamma\ 1.0\ EQ_Z}$$

where $\boldsymbol{\psi_{2i}}$ is obtained from the following table.

Table A1.1 - Recommended values of ψ factors for buildings

Action	ψ_0	ψ_1	ψ_2
Imposed loads in buildings, category (see EN 1991-1-1)			
Category A : domestic, residential areas	0,7	0,5	0,3
Category B : office areas	0,7	0,5	0,3
Category C : congregation areas	0,7	0,7	0,6
Category D : shopping areas	0,7	0,7	0,6
Category E : storage areas	1,0	0,9	0,8
Category F : traffic area, vehicle weight ≤ 30kN	0,7	0,7	0,6
Category G : traffic area, 30kN < vehicle weight ≤ 160kN	0,7	0,5	0,3
Category H : roofs	0	0	0
Snow loads on buildings (see EN 1991-1-3)*			
Finland, Iceland, Norway, Sweden	0,70	0,50	0,20
Remainder of CEN Member States, for sites located at altitude H > 1000 m a.s.l.	0,70	0,50	0,20
Remainder of CEN Member States, for sites located at altitude H ≤ 1000 m a.s.l.	0,50	0,20	0
Wind loads on buildings (see EN 1991-1-4)	0,6	0,2	0
Temperature (non-fire) in buildings (see EN 1991-1-5)	0,6	0,5	0
NOTE The ψ values may be set by the National annex. * For countries not mentioned below, see relevant local conditions.			

EQ refers to the inelastic response spectrum. The enveloped results are employed to design the elements to its **ultimate state plastic capacity**.

The EC8 limits the **story drift** in a structure. The inelastic displacement drift is obtained by multiplying the drift obtained from using the inelastic response spectrum by the behaviour factor. This value should not exceed 0.4% of the height of the story, H.

$$\Delta_d = q.\Delta_e < 0.004H\ \nu$$

where the reduction factor ν (accounting for the lower return period of the seismic event associated with the serviceability limit state) is

Table 4.1: Values of the reduction factor ν

Importance category	I	II	III	IV
Reduction Factor ν	[2,5]	[2,5]	[2,0]	[2,0]

Second order effects (P-Δ effects) need not be considered if the following condition is met for all storeys.

$$\theta = \frac{P_{tot} \cdot d_r}{V_{tot} \cdot h} \leq 0,10$$

where

θ interstorey drift sensitivity coefficient,

P_{tot} total gravity load at and above the storey considered, in accordance with the assumptions made for the computation of the seismic action effects.

d_r design interstorey drift, evaluated as the difference of the average lateral displacements at the top and bottom of the storey under consideration and calculated according to 3.4,

V_{tot} total seismic storey shear,

h interstorey height.

If $0.1 < \theta < 0.2$, then P-Δ effects can be accounted for approximately by increasing seismic actions by $1/(1-\theta)$. The value of θ shall not exceed 0.3.

1.1.5.2.6 IBC 2000 Elastic and Inelastic Response Spectrum

The IBC spectrum is defined by three lines governing low period response, mid period response and high period response. The response spectrum $S_a(g)(T)$ is defined

$$S_a = 0.6\frac{S_{DS}}{T_0}T + 0.4S_{DS} \qquad \text{for} \qquad 0 < T < T_0$$

$$S_a = S_{DS} \qquad \text{for} \qquad T < T_S$$

$$S_a = \frac{S_{D1}}{T} \qquad \text{for} \qquad T > T_S$$

where

$$T_0 = 0.2\,S_{D1} / S_{DS}$$

$$T_S = S_{D1} / S_{DS}$$

where the design spectral response acceleration parameters

$$S_{DS} = \tfrac{2}{3} S_{MS}$$

$$S_{D1} = \tfrac{2}{3} S_{M1}$$

and where the maximum elastic spectra are

$$S_{MS} = F_a S_S$$

$$S_{M1} = F_v S_1$$

where S_S and S_1 are the short period acceleration response parameter and the 1s period acceleration response parameter and the coefficients F_a and F_v which are dependent upon the site class and S_S/S_1 are defined

Seismic coefficients F_a

Site Class	Mapped spectral response at short periods, S_S				
	$S_S \le 0.25$	$S_S = 0.50$	$S_S = 0.75$	$S_S = 1.00$	$S_S \ge 1.25$
A	0.8	0.8	0.8	0.8	0.8
B	1.0	1.0	1.0	1.0	1.0
C	1.2	1.2	1.1	1.0	1.0
D	1.6	1.4	1.2	1.1	1.0
E	2.5	1.7	1.2	0.9	as F
F	Specialist geotechnical advice required				

Seismic coefficients F_v

Site Class	Mapped spectral response at 1s periods, S_1				
	$S_S \le 0.1$	$S_S = 0.2$	$S_S = 0.3$	$S_S = 0.4$	$S_S \ge 0.5$
A	0.8	0.8	0.8	0.8	0.8
B	1.0	1.0	1.0	1.0	1.0
C	1.7	1.6	1.5	1.4	1.3
D	2.4	2.0	1.8	1.6	1.5
E	3.5	3.2	2.8	2.4	as F
F	Specialist geotechnical advice required				

The short period force reduction should normally be ignored in response spectrum calculations.

1.1.5.2.7 FEMA 356 Elastic and Inelastic Response Spectrum

The FEMA 356 spectrum is defined by three lines governing low period response, mid period response and high period response. The response spectrum $S_a(g)(T)$ is defined

$$S_a = S_{XS}\left[\left(\frac{5}{B_S}-2\right)\frac{T}{T_S}+0.4\right] \quad \text{for} \quad 0<T<T_0$$

$$S_a = S_{XS}/B_S \quad \text{for} \quad T<T_S$$

$$S_a = \left(S_{X1}/(B_1 T)\right) \quad \text{for} \quad T>T_S$$

where

$$T_S = (S_{X1}B_S)/(S_{XS}B_1)$$

$$T_0 = 0.2T_S$$

where

$$S_{XS} = F_a S_S$$

$$S_{X1} = F_v S_1$$

where S_S and S_1 are the short period acceleration response parameter and the 1s period acceleration response parameter and the coefficients F_a and F_v which are dependent upon the site class and S_S/S_1 are defined

Seismic coefficients F_a

Site Class	Mapped spectral response at short periods, S_S				
	$S_S \le 0.25$	$S_S = 0.50$	$S_S = 0.75$	$S_S = 1.00$	$S_S \ge 1.25$
A	0.8	0.8	0.8	0.8	0.8
B	1.0	1.0	1.0	1.0	1.0
C	1.2	1.2	1.1	1.0	1.0
D	1.6	1.4	1.2	1.1	1.0
E	2.5	1.7	1.2	0.9	as F
F	Specialist geotechnical advice required				

Seismic coefficients F_v

Site Class	Mapped spectral response at 1s periods, S_1				
	$S_S \le 0.1$	$S_S = 0.2$	$S_S = 0.3$	$S_S = 0.4$	$S_S \ge 0.5$
A	0.8	0.8	0.8	0.8	0.8
B	1.0	1.0	1.0	1.0	1.0
C	1.7	1.6	1.5	1.4	1.3
D	2.4	2.0	1.8	1.6	1.5
E	3.5	3.2	2.8	2.4	as F
F	Specialist geotechnical advice required				

Site class D is default. The values for B_S and B_1 are a function of the effective viscous damping (β). Default value for β is 5%.

Damping coefficients B_S and B_1

Effective Viscous Damping (β) %	Damping coefficients	
	B_S	B_1
≤ 2	0.8	0.8
5	1.0	1.0
10	1.3	1.2
20	1.8	1.5
30	2.3	1.7
40	2.7	1.9
≥ 50	3.0	2.0

The vertical response is 2/3 of the above response spectrum.

1.1.5.3 GL, ML SDOF Response Spectrum Analysis – Equivalent Lateral Static Force Method

The equivalent lateral force method is a special case of the multi-modal approach, i.e. by only including the fundamental mode. A SDOF system subject to base excitations, in relative terms has the following ODE.

$$m\ddot{u}_r(t) + c\dot{u}_r(t) + ku_r(t) = -m\ddot{u}_s(t)$$

The response will be obtained in relative terms as follows from Duhamel's (Convolution) Integral.

$$\begin{aligned}
u_r(\tau = t) &= \frac{1}{m\omega_d}\int_{\tau=0}^{\tau=t} p(\tau)e^{-\xi\omega_n(t-\tau)}\sin\omega_d(t-\tau)d\tau \\
&= \frac{1}{m\omega_d}\int_{\tau=0}^{\tau=t} -m\ddot{u}_s(\tau)e^{-\xi\omega_n(t-\tau)}\sin\omega_d(t-\tau)d\tau \\
&= \frac{1}{\omega_d}\int_{\tau=0}^{\tau=t} -\ddot{u}_s(\tau)e^{-\xi\omega_n(t-\tau)}\sin\omega_d(t-\tau)d\tau \\
u_{r\max}(\tau = t) &= \frac{1}{\omega_d}\text{MAX}\left[\int_{\tau=0}^{\tau=t}\ddot{u}_s(\tau)e^{-\xi\omega_n(t-\tau)}\sin\omega_d(t-\tau)d\tau\right] \\
&= \frac{1}{\omega_d}S_v \qquad \text{where } S_v = \text{pseudo spectral velocity}
\end{aligned}$$

Because of the irregularity of the excitation function, the Duhamel's Integral will have to be solved numerically. The plot of the maximum dynamic displacement response u_{rmax} for various natural periods T_i is known as the spectral displacement (or the displacement response spectrum), S_D. The spectral velocity (or the velocity response spectrum) is S_V and the spectral acceleration is S_A. These are related *approximately* as follows.

$$S_V = \omega_{n,d}S_D = \frac{S_A}{\omega_{n,d}}$$

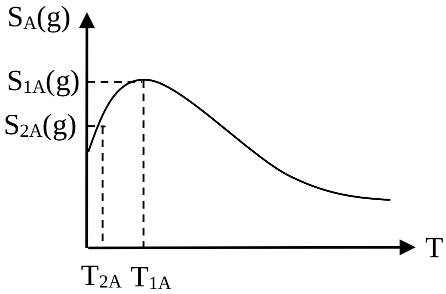
It is a mere coincidence that these relationships are same as that between acceleration, velocity and displacement in simple harmonic motion. The derivation of the response spectrum enables the response prediction of SDOF systems. For a SDOF system, the maximum dynamic force (i.e. the base shear) on the SDOF mass is then

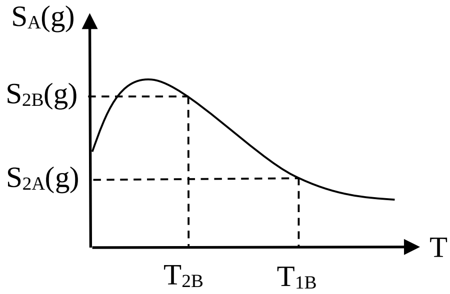
$$\begin{aligned}
\text{Max dynamic force} = ku_{rmax} &= k\frac{S_V}{\omega_d} = m\omega_n^2\frac{S_V}{\omega_d} \\
&\approx m\omega_n S_V \\
&\approx \frac{W}{g}S_A \\
&\approx WS_A(g)
\end{aligned}$$

If the structure is idealized as a SDOF system characterized by its first fundamental frequency or period T_1, the response is computed as follows

(i) The fundamental period T_1 of the structure corresponding to the horizontal first mode is established
(ii) The response is computed in g ms^{-2} from the response spectrum $S_A(g)(T_1)$
(iii) The base shear is computed as $F_{base\ shear} = S_A(g)(T_1)W$
(iv) The base shear is distributed linearly along the height of the structure and a static analysis performed; this effectively determines the equivalent dynamic response

Note that in the equation for base shear $V = S_A(g)W$, the acceleration spectrum $S_A(g)$ is in terms of g. Also, W is the modal weight, i.e. the modal mass times g. Here in the equivalent static force method, we only consider the fundamental period. Noting for most structures, the fundamental modal weight W_1 is some 80-90% of the total weight, if the spectral ordinates S_{A1} is larger or equal to S_{A2}, S_{A3} etc …, then replacing W_1 by W_T will always yield an upper bound on V.





Since S_{1A} is multiplied by W_1, which is normally much larger than W_2, W_3 etc …, then multiplying S_{1A} by W_T will yield an upper bound on the total force. This is clearly not the case for system B.

The equivalent lateral static force method is very crude since it **does not consider higher (than the fundamental) modes** of vibration, and hence is not really a dynamic analysis. Higher modes may be significant especially at the local level even if they do not contribute much to the base shear. This is simply because higher modes have shapes which are fundamentally different from that of the fundamental mode cause **localized higher forces**. Higher modes can be addressed by performing the multi-modal method.

We have said that the base shear is $V = W_T S_A(g)$. Codes of practice generally provide methods of calculating this expression. This expression then of course account for the fundamental mode only, but the response spectra derived from codes of practice can be used for the multi-modal spectrum analysis. Now, for this fundamental mode approach (equivalent lateral load), codes of practice state $V = S_A(g)W_T$. Typically $S_A(g)$ depends on

- I. Period dependent coefficient representing the shape of the spectrum
- II. Importance factor representing the probability of the design load being exceeded
- III. Zone factor representing the seismic hazard at the site
- IV. Site response parameter representing the amplification characteristic of the soil
- V. Response modification factor representing the ability of the structure to dissipate energy

1.1.5.3.1 UBC 1997

For Seismic Zones 3 and 4, the static load procedure is only applicable to regular structures under 240 ft. in height or irregular structures not more than five stories or 65 ft. in height. The equivalent lateral force

$$V = 0.11 C_a I W \quad < \quad V = \frac{C_v I}{RT} W \quad < \quad V = \frac{2.5 C_a I}{R} W$$

W is the **total weight** of the structure. The UBC further requires that for Seismic Zone 4,

$$V \geq \frac{0.8 Z N_v I}{R} W$$

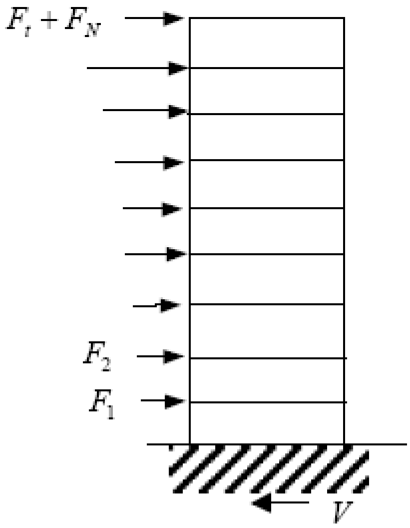
I is the structural importance factor.

Occupancy Category	Essential Facilities	Hazardous Facilities	Special Occupancy Structures	Standard Occupancy Structures
I	1.25	1.25	1.00	1.00

The base shear is distributed based on

$$F_i = \frac{(V - F_t) W_i h_i}{\sum_{j=1}^{N} W_j h_j}$$

which is based on the assumption that the fundamental mode of the structure has a linear shape. F_t is the top storey force to account for higher mode effects. The force at each level is proportional to the height and story weight.



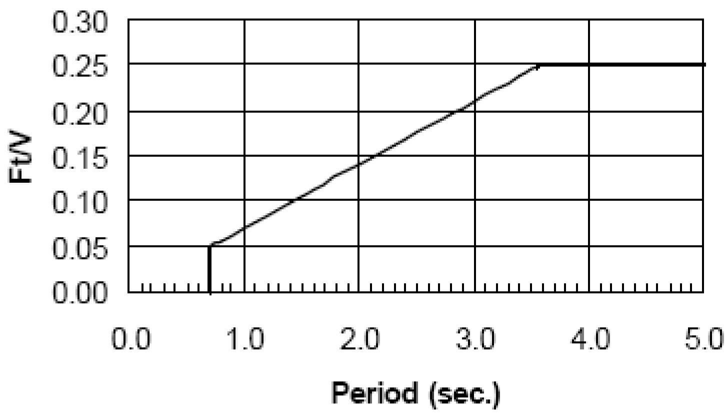
Clearly, the total base shear is

$$V = F_t + \sum_{i=1}^{N} F_i$$

In general, higher-mode effects are only important for structures that have relatively long fundamental periods. Hence, in the UBC, F_t is zero when T < 0.7 seconds and it increases in proportion to T up to 0.25V, i.e.

$$F_t = 0.07TV \leq 0.25V \quad \text{for } T > 0.7 \text{ sec.}$$

$$F_t = 0 \quad \text{for } T \leq 0.7 \text{ sec.}$$



The fundamental period T can be obtained using the empirical formula where h_N is the height of the building **in feet** as follows

$$T = C_t (h_N)^{3/4}$$

$C_t = 0.035$ for steel moment-resisting frames

$C_t = 0.030$ for reinforced concrete moment-resisting frames and eccentrically braced frames

$C_t = 0.020$ for all other buildings

Alternatively, it can be obtained using modal analysis or Rayleigh's formula

$$T_1 = 2\pi\sqrt{\frac{1}{g}\frac{\sum W_i \delta_i^2}{\sum W_i \delta_i}}$$

Although the latter method is more rational, the UBC requires that the value of T obtained with the modal analysis or Rayleigh's method be no greater than that obtained with the empirical by more than 30% for Seismic Zone 4 and 40% for Seismic Zones 1, 2, and 3.

1.1.5.3.2 EC8

Base shear, $F_b = S_d(T_1).W$

Fundamental period of vibration, $T_1 = C_t H^{0.75}$. Alternatively, it can be obtained using modal analysis or Rayleigh's formula

$$T_1 = 2\pi\sqrt{\frac{1}{g}\frac{\sum W_i \delta_i^2}{\sum W_i \delta_i}}$$

The base shear is assumed linearly distributed along the height of the building, hence the seismic forces at level i are given by F_i where z_i is the height of the storey from the ground.

$$F_i = F_b \cdot \frac{z_i . W_i}{\sum z_j . W_j}$$

1.1.5.3.3 IBC 2000

The base shear is

$$V = C_s W$$

where

$$C_s = 0.044 S_{DS} I_E \quad < \quad C_s = \frac{S_{DS}}{(R/I_E)} \quad < \quad C_s = \frac{S_{D1}}{(R/I_E)T}$$

For buildings in Seismic Design Category E or F where $S_1 > 0.6g$, C_s must be greater than

$$C_s = \frac{0.5 S_1}{(R/I_E)}$$

The fundamental period of the structure is calculated as

$$T_a = C_T h_n^{3/4}$$

where the factor C_T depends on the type of building frame and h_N is the height of the building in feet. Alternatively the fundamental period of the building can be calculated from a modal analysis, but cannot exceed

$$T = C_u T_a$$

where C_u is a function of S_{D1} (the design response acceleration at 1 second period).

The vertical force is distributed as follows.

$$F_x = C_{vx} V$$

where

$$C_{vx} = \frac{w_x h_x^k}{\sum_{i=1}^{n} w_i h_i^k}$$

where the exponent k depends on the building period from 1 for buildings with a short period to 2 for buildings with a long period. F_x is the force applied at each storey, this is then distributed across the nodes on the storey in proportion to the nodal masses.

1.1.5.3.4 FEMA 356

The base shear is given by

$$V = C_1 C_2 C_3 C_m S_a W$$

The fundamental period is calculated from

$$T_a = C_T h_n^{\beta}$$

The factors C_T and β depend on the type of building frame and h is the height of the building in feet. Alternatively, the fundamental period can be calculated from modal analysis.

The vertical distribution of forces is given by

$$F_x = C_{vx} V$$

where

$$C_{vx} = \frac{w_x h_x^k}{\sum_{i=1}^{n} w_i h_i^k}$$

where the exponent k depends on the building period from 1 for buildings with a short period to 2 for buildings with a long period. F_x is the force applied at each storey, this is then distributed across the nodes on the storey in proportion to the nodal masses.

1.1.5.4 GL, ML MDOF Response Spectrum Analysis – Multi-Modal Seismic Analysis

For a MDOF system, the equation of motion for support excitation in relative terms is

$$[M]\{\ddot{u}(t)\} + [C]\{\dot{u}(t)\} + [K]\{u(t)\} = -[M]\{1\}\ddot{u}_s(t)$$

Note that {D} = {1} is a unity vector corresponding to the rigid body motion. The support excitation causes all the masses to produce an inertial force opposing its motion (D'Alembert's Principle).

Let $\{u(t)\} = [\Phi]\{\xi(t)\}$

$$[M][\Phi]\{\ddot{\xi}(t)\} + [C][\Phi]\{\dot{\xi}(t)\} + [K][\Phi]\{\xi(t)\} = -[M]\{1\}\ddot{u}_s(t)$$

Premultiplying by $[\Phi]^T$ reduces the coupled system of ODEs to a system of uncoupled ODEs

$$[\Phi]^T[M][\Phi]\{\ddot{\xi}(t)\} + [\Phi]^T[C][\Phi]\{\dot{\xi}(t)\} + [\Phi]^T[K][\Phi]\{\xi(t)\} = -[\Phi]^T[M]\{1\}\ddot{u}_s(t)$$

$$[\mathsf{M}]\{\ddot{\xi}(t)\} + [\mathsf{C}]\{\dot{\xi}(t)\} + [\mathsf{K}]\{\xi(t)\} = -[\Phi]^T[M]\{1\}\ddot{u}_s(t)$$

Each uncoupled equation being

$$\mathsf{M}_i\ddot{\xi}_i(t) + \mathsf{C}_i\dot{\xi}_i(t) + \mathsf{K}_i\xi_i(t) = -\{\phi_i\}^T[M]\{1\}\ddot{u}_s(t)$$

Dividing the equation of motion for the ith natural mode from above by its modal mass gives

$$\mathsf{M}_i\ddot{\xi}_i(t) + \mathsf{C}_i\dot{\xi}_i(t) + \mathsf{K}_i\xi_i(t) = -\{\phi_i\}^T[M]\{1\}\ddot{u}_s(t)$$

$$\ddot{\xi}_i(t) + \frac{2\xi_i\mathsf{M}_i\omega_{ni}}{\mathsf{M}_i}\dot{\xi}_i(t) + \frac{\mathsf{K}_i}{\mathsf{M}_i}\xi_i(t) = \frac{-\{\phi_i\}^T[M]\{1\}\ddot{u}_s(t)}{\mathsf{M}_i}$$

$$\ddot{\xi}_i(t) + 2\xi_i\omega_{ni}\dot{\xi}_i(t) + \omega_{ni}^2\xi_i(t) = \Gamma_i\ddot{u}_s(t)$$

where the ith mode participation factor is defined as

$$\Gamma_i = -\frac{\{\phi_i\}^T[M]\{1\}}{\mathsf{M}_i} = -\frac{\{\phi_i\}^T[M]\{1\}}{\{\phi_i\}^T[M]\{\phi_i\}}$$

Bereft of the modal participation factor, the equations of motion reduce to simple SDOF systems characterized by only the natural frequency and damping and subject to the excitation $\ddot{u}_s(t)$. Duhamel's Integral is used to evaluate numerically to derive the response spectra to $\ddot{u}_s(t)$. That is to say, for

$$\ddot{q}_i(t) + 2\xi_i\omega_{ni}\dot{q}_i(t) + \omega_{ni}^2 q_i(t) = \ddot{u}_s(t)$$

the response is

$$q_r(\tau = t) = \frac{1}{m\omega_d}\int_{\tau=0}^{\tau=t} p(\tau)e^{-\xi\omega_n(t-\tau)}\sin\omega_d(t-\tau)d\tau$$

$$= \frac{1}{m\omega_d}\int_{\tau=0}^{\tau=t} -m\ddot{u}_s(\tau)e^{-\xi\omega_n(t-\tau)}\sin\omega_d(t-\tau)d\tau$$

$$= \frac{1}{\omega_d}\int_{\tau=0}^{\tau=t} -\ddot{u}_s(\tau)e^{-\xi\omega_n(t-\tau)}\sin\omega_d(t-\tau)d\tau$$

$$q_{r\,max}(\tau = t) = \frac{1}{\omega_d}\mathrm{MAX}\left[\int_{\tau=0}^{\tau=t}\ddot{u}_s(\tau)e^{-\xi\omega_n(t-\tau)}\sin\omega_d(t-\tau)d\tau\right]$$

$$= \frac{1}{\omega_d}S_v \qquad \text{where } S_v = \text{pseudo spectral velocity}$$

The plot of the maximum dynamic displacement response for various natural periods T_i is known as the spectral displacement (or the displacement response spectrum), S_D i.e. q_{rmax}. The spectral velocity (or the velocity response spectrum) is S_V and the spectral acceleration is S_A. These are related *approximately* as follows.

$$S_V = \omega_{n,d} S_D = \frac{S_A}{\omega_{n,d}}$$

It is a mere coincidence that these relationships are same as that between acceleration, velocity and displacement in simple harmonic motion. The **maximum** modal responses are then

$$\xi_{i\,max}(t) = \Gamma_i S_D$$
$$\dot{\xi}_{i\,max}(t) = \Gamma_i S_V$$
$$\ddot{\xi}_{i\,max}(t) = \Gamma_i S_A$$

Once the individual maximum modal responses are computed, the maximum physical responses is calculated from the following based on some method of superposition.

$$\{u_{rmax}(t)\} = [\Phi]\{\xi_{max}(t)\}$$
$$\{\dot{u}_{rmax}(t)\} = [\Phi]\{\dot{\xi}_{max}(t)\}$$
$$\{\ddot{u}_{rmax}(t)\} = [\Phi]\{\ddot{\xi}_{max}(t)\}$$

Thus, if the structure is idealized as a MDOF system characterized by its first few frequencies or period T_1, the response spectrum analysis method is performed as follows.

(i) The response spectrum is first established from the forcing acceleration time history. The acceleration response of a notional mode (of a particular natural frequency) is established separately from solving the equation of motion bereft of the modal participation factor

$$\ddot{q}_i(t) + 2\xi_i \omega_{ni} \dot{q}_i(t) + \omega_{ni}^2 q_i(t) = \ddot{u}_s(t)$$

resulting in the solution for each notional mode (each natural frequency) in the time domain by Duhamel's Integral

$$q_r(\tau = t) = \frac{1}{\omega_d} \int_{\tau=0}^{\tau=t} -\ddot{u}_s(\tau) e^{-\xi\omega_n(t-\tau)} \sin\omega_d(t-\tau)d\tau$$

the maximum of which is found for each notional mode (each natural frequency) to define the (maximum) response spectra

$$S_D = q_{r\,max}(\tau = t) = \frac{1}{\omega_d} \mathrm{MAX}\left[\int_{\tau=0}^{\tau=t} \ddot{u}_s(\tau) e^{-\xi\omega_n(t-\tau)} \sin\omega_d(t-\tau)d\tau\right]$$

$$S_V = \omega_{n,d} S_D = \frac{S_A}{\omega_{n,d}}$$

(ii) The modal (or generalized) mass M_i is computed for each mode i as follows

$$M_i = \{\phi_i\}^T[M]\{\phi_i\}$$

The actual magnitudes of the modal masses in itself are incomplete to describe the importance of the mode (as the amplitude of the modal force must also be considered) as they depend on the method of eigenvector normalization.

(iii) The amplitude of the modal force is for each mode i

$$L_i = \{\phi_i\}^T[M]\{1\}$$

where the vector {D} = {1} simply represents the vector of rigid body motion.

(iv) And the modal participation factor and effective mass for each mode i is defined as

Modal participation factor, $\Gamma_i = L_i / M_i$

Modal effective mass, $EffectiveMass_i = L_i^2 / M_i$

both representing the relative contribution of each mode (excluding the frequency content of the excitation function which is encapsulated in the response spectrum) to the response prior to multiplication by the input function. The fundamental mode shape will usually have a dominant participation factor and effective mass associated with it. Higher modes will have a lower participation because the positive and negative components of the mode shape in the participation expression cancels out. This generally does not occur in load excitations unless the loading is uniformly distributed, such as is the case in enforced motion. Sufficient modes must be included such that $\sum L_i^2/M_i$ is at least 90% of the total mass. Participation factor and modal effective mass is calculated in NASTRAN using the Case Control Command MEFFMASS.

(v) The **maximum** modal response in modal space for mode i is computed as follows

$$\xi_{i\max}(t) = \Gamma_i S_D$$
$$\dot{\xi}_{i\max}(t) = \Gamma_i S_V$$
$$\ddot{\xi}_{i\max}(t) = \Gamma_i S_A$$

(vi) The **maximum** modal responses in physical space for mode i is then computed as follows

$$\{u_{i\max}(t)\} = \{\phi_i\}\xi_{i\max}(t) = \{\phi_i\}\Gamma_i S_D$$
$$\{\dot{u}_{i\max}(t)\} = \{\phi_i\}\dot{\xi}_{i\max}(t) = \{\phi_i\}\Gamma_i S_V$$
$$\{\ddot{u}_{i\max}(t)\} = \{\phi_i\}\ddot{\xi}_{i\max}(t) = \{\phi_i\}\Gamma_i S_A$$

(vii) The (dynamic) base shear for a particular mode i is

$$V_i = EffectiveMass_i S_A$$
$$= \frac{\left[\{\phi_i\}^T[M]\{1\}\right]^2}{\{\phi_i\}^T[M]\{\phi_i\}} S_A$$

If S_A is expressed as a fraction of g, then the modal base shear is

$$V_i = EffectiveWeight_i S_A(g)$$
$$= \frac{\left[\{\phi_i\}^T[W]\{1\}\right]^2}{\{\phi_i\}^T[W]\{\phi_i\}} S_A(g)$$

Hence, for a simple building of floors denoted by m, the modal base shear for mode i is

$$W_i = \frac{\left(\sum_{m=1}^{m=all} w_m \phi_{mi}\right)^2}{\sum_{m=1}^{m=all} w_m \phi_{mi}^2} S_A(g)$$

(viii) Finally the modal force response (i.e. force at every DOF) for mode i is computed as follows

$$
\begin{aligned}
\{F_i\} &= [M]\{\ddot{u}_i(t)\} \\
&= [M]\{\phi_i\}\Gamma_i S_A \\
&= [M]\{\phi_i\}\frac{\{\phi_i\}^T[M]\{1\}}{\{\phi_i\}^T[M]\{\phi_i\}} S_A \\
&= \frac{[M]\{\phi_i\}}{\{\phi_i\}^T[M]\{1\}} \frac{\left[\{\phi_i\}^T[M]\{1\}\right]^2}{\{\phi_i\}^T[M]\{\phi_i\}} S_A \\
&= \frac{[M]\{\phi_i\}}{\{\phi_i\}^T[M]\{1\}} V_i
\end{aligned}
$$

The above can also be expressed in weight ratios as

$$\{F_i\} = \frac{[W]\{\phi_i\}}{\{\phi_i\}^T[W]\{1\}} V_i$$

Hence, for a simple building of floors denoted by m, the force at floor m due to mode i is

$$f_{mi} = V_i \frac{M_m \phi_{mi}}{\sum_{m=1}^{m=all} M_m \phi_{mi}} = V_i \frac{w_m \phi_{mi}}{\sum_{m=1}^{m=all} w_m \phi_{mi}}$$

(ix) The modal responses are then combined using the SRSS or the CQC combinations. The upper bound responses of the different modes can be established using the SRSS method if the modal frequencies are well separated. Alternatively, the complete quadratic combination (CQC) method should be used if some modal frequencies are closely spaced, in which case the absolute sum of the response of these closely spaced modes is taken on the assumption that they can peak in phase with each other. These combinations can be used for any modal response, be it the displacement, velocity, acceleration, modal force responses or modal base shears.

$$\text{SRSS}: V = \sqrt{\sum_{i=1}^{n} V_i^2}$$

$$\text{CQC}: V = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} V_i \beta_{ij} V_j} \qquad \beta_{ij} = \frac{8(\zeta_i \zeta_j)^{1/2}(\zeta_i + r\zeta_j) r^{1.5}}{(1-r^2)^2 + 4\zeta_i\zeta_j r(1+r^2) + 4(\zeta_i^2 + \zeta_j^2) r^2}$$

The cross-modal parameters β_{ij} are a function of the duration and frequency content of the earthquake record, and frequencies and damping characteristics of the structure. For an earthquake duration much longer than the periods of the structure and a response spectrum which is not exceptionally spiky and constant modal damping, the cross-modal parameters are given by

$$\beta_{ij} = \frac{8\zeta^2(1+r) r^{1.5}}{(1-r^2)^2 + 4\zeta^2 r(1+r)^2}$$

where ζ is the damping ratio and r is the period ratio (T_i/T_j). Indeed, the use of CQC is recommended in general, but the difference between the two methods diminishes rapidly as the mode spacing increases.

The assumptions and limitations of a multi-modal response spectrum analysis are as follows: -

(i) The response spectrum method only provides the **maximum values** of response.

(ii) The analysis is **geometrically and materially linear** and the nonlinearity is very crudely incorporated into the analysis using the **q-factor** (Europe) to reduce the amplitudes of the elastic response spectra to an inelastic response spectra. Higher modes are accounted for, i.e. the inter-storey forces generated due to higher modes are taken into account by computing the modal displacement responses and converting that to the modal force responses. These are then applied as static forces in a linear static analysis. We can say that the dynamic effects of higher modes are effectively taken into account, however **only if the response is linear and elastic**. An alternative would be to perform nonlinear time domain analyses of appropriately scaled (to match response spectra) seismic time histories and envelope the results.

(iii) Since the response spectrum is the maximum response of a series of SDOF systems, it must be remembered that the peak response for one SDOF system does not occur at the same time as the peak response for another. In other words there is **no phase information between the responses of the individual modes** or SDOF systems. Hence, various methods to combine these peak responses are employed such as the SRSS (Square Root of the Sum of the Squares), the ABS (absolute values) the NRL (U.S. Navy Shock Design Modal Summation) or the CQC (complete quadratic combination). Thus, the results of a response spectrum analysis are not a set of results in equilibrium but the maximum values that can be expected.

(iv) There is also **no consideration of phase of the response with respect to the loading** as only the magnitude is computed.

(v) Another restriction of the response spectrum method is that it is **only valid if all support motion in a given direction is identical**. This is usually valid for ground movements applied to buildings or local floor movements applied to equipment but it is not applicable to piping systems where the anchor points of the pipe are connected to different structures or even different parts of the same structure [2]. Another case would be a long span suspension bridge where the earthquake excitations may differ on either side of the abutment.

(vi) It is assumed that each oscillator mass is very small relative to the base structural mass so that the oscillator does not influence the dynamic behavior of the base structure, i.e. **no soil-structure interaction** accounted for in the earthquake vibration analysis.

If **base isolation** is employed, the stiffness of the base isolator can be incorporated into the finite element model, hence elongating the periods. Base isolation in effect simply introduces a low frequency component into the system. The periods will be elongated and the corresponding response spectra value will be lower. Base isolation is thus used for stiff structures in order to elongate the effective period of the system.

[2] NAFEMS. *A Finite Element Primer.* NAFEMS Ltd., Great Britain, 1992.

1.1.5.4.1 UBC 1997

The design base shear determined from the multi modal response shall not be less than 90% of that determined by the equivalent static load procedure for regular structures and 100% for irregular structures. Hence there is a requirement for a scale factor for the force as follows

$$\text{Force scale factor} = \alpha_T \frac{V_{\text{equivalent static}}}{V_{\text{multi-modal}}} \quad \text{but} \quad \text{Force scale factor} > \frac{1}{R}$$

where α_T is 0.9 for regular structures or 1.0 for irregular structures. Likewise the scale factor for displacements

$$\text{Displacement scale factor} = \alpha_T \frac{V_{\text{equivalent static}}}{V_{\text{multi-modal}}}$$

1.1.5.4.2 EC8

No additional specific requirements.

1.1.5.4.3 IBC 2000

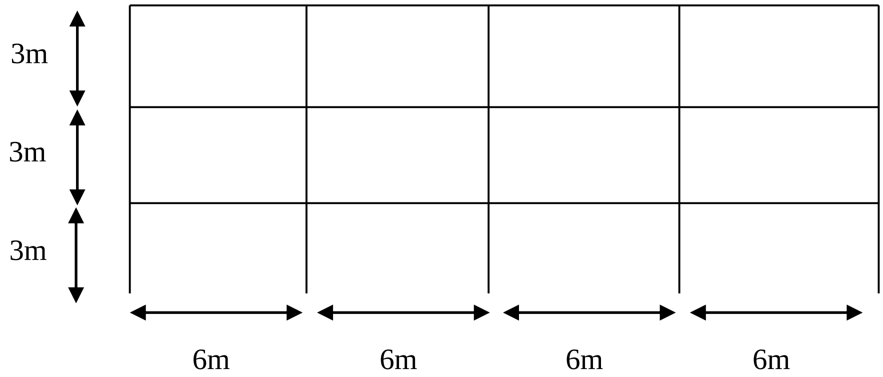
No additional specific requirements.

1.1.5.4.4 FEMA 356

No additional specific requirements.

1.1.5.5 Example Application EC8

Following a seismic hazard of **Section 1.1.1.18** and the soil response analyses of **Section 1.1.2.15.1**, a peak ground acceleration at the bedrock level of 0.6g and at the soil level of 0.15g has been obtained. The results of these two initial steps culminate in the response spectra. The 0.15g acceleration corresponds to the value of the response spectrum at period, T = 0.0s. The soil analysis will also produce ordinates of the response spectrum for other values of the period, T. With this, the seismic action effects on the structure are determined based on the simplified equivalent lateral force analysis and the multi-modal response spectrum analyses. The level of member detailing corresponds to ductility class "M" and the site soil conditions are such that soil class "B" is appropriate. The regularity of the structure in plan enables the analysis to be based on a typical frame as shown below. The floor slabs are assumed to be infinitely rigid (i.e. no large openings etc.) in order to distribute the floor shear forces amongst the columns in diaphragm action.



1.1.5.5.1 Structural Loading

Assume a concrete volumetric weight of 25 kN/m^3.

Loads on Slabs

Self-weight of slab + partitions, G = 6 kN/m^2
Live load, Q = 3 kN/m^2

Loads on Beams

Dead load on beam, G_k = self-weight + slab dead load x width = 0.3 x 0.5 x 25 + 6 x 5 = 33.75 kN/m
Live load on beam, Q_k = slab live load x width = 3 x 5 = 15 kN/m

1.1.5.5.2 Seismic Forces Using the Simplified Equivalent Lateral Force Analysis of EC8

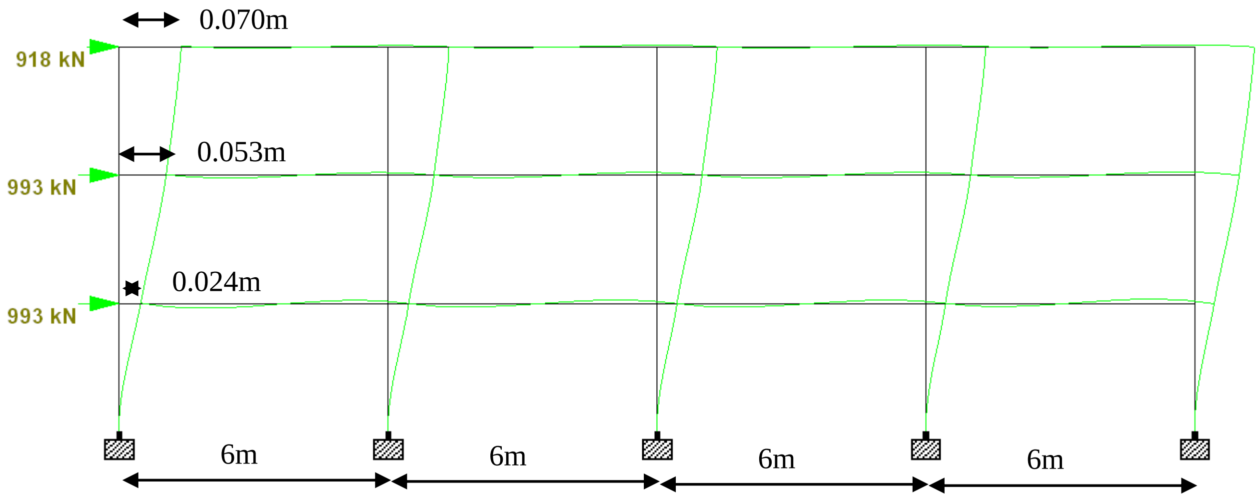
Fundamental Period of Vibration Using Simplified Method

Fundamental period of vibration, $T_1 = C_tH^{0.75} = 0.075 \text{ x } H^{0.75} = 0.075 \text{ x } 9^{0.75} = 0.39$s

Fundamental Period of Vibration Using Rayleigh's Method

Seismic self weight on storey 3: $G_k + \Psi_E Q_k$ = 33.75 x 24 + 0.3 x (15 x 24) = 918 kN (no column)
Seismic self weight on storey 2: $G_k + \Psi_E Q_k$ = 918 + 5 x 0.4 x 0.5 x 3 x 25 = 993 kN
Seismic self weight on storey 1: $G_k + \Psi_E Q_k$ = 993 kN
A structural static analysis programme was used to give the displacement of the structure under the seismic weight as shown in below.

Cross-sectional Properties	Beam	Column
Area	bd = 30 x 50 = 1500 cm^2	bd = 50 x 40 = 2000 cm^2
Second moment of area about major axis	$1/12bd^3$ = 1/12 x 30 x 50^3 = 312500 cm^4	$1/12bd^3$ = 1/12 x 40 x 50^3 = 416667 cm^4
Second moment of area about minor axis	$1/12db^3$ = 1/12 x 50 x 30^3 = 112500 cm^4	$1/12db^3$ = 1/12 x 50 x 40^3 = 266667 cm^4



$$T_1 = 2\pi\sqrt{\frac{1}{g}\frac{\sum W_i\delta_i^2}{\sum W_i\delta_i}} = 2\pi\sqrt{\frac{1}{9.81}\left(\frac{918x0.070^2 + 993x0.053^2 + 993x0.024^2}{918x0.070 + 993x0.053 + 993x0.024}\right)} = 0.47s$$

The fundamental period of the structure calculated from both the simplified method and Rayleigh's method are very similar. This is because the structure is uniform and regular. The low period also signifies a relatively stiff structure, which is why the simplified method provided an accurate estimate.

Behaviour Factor

The behaviour factor represents the ability of the structure to respond in a ductile manner. Higher values of q require greater level of detailing for ductility. However, the greater energy absorbing capacity provided by the greater ductility enables the structure to be designed for lower seismic action forces. The behaviour factor q reduces as T increases, but dependence on the period is not explicitly considered. Also, over-strength and redundancy parameters are not taken into account either.

Behaviour factor, q = q_0 k_D k_R k_W
where basic behaviour factor, q_0 = 5.0 as this is a RC frame system
k_D = 0.75 as this is a ductility class "M" structure
k_R = 1.0 as this is a regular structure
k_W = 1.0 as this is a frame system
Hence, q = 5 x 0.75 x 1 x 1 = 3.75

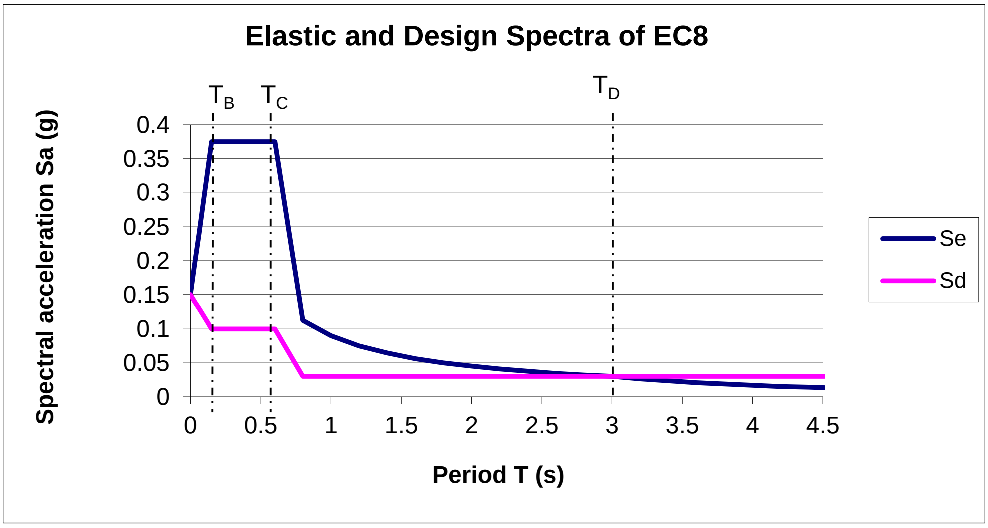
Inelastic Spectral Acceleration of EC8

From previous soil response analysis of **Section 1.1.2.15.1**,

(i) the soil at the site is characterized by shear wave velocities of 200 to 250 m/s at depths of 10m and 450 to 500 at depths of 50m. This signifies Soil Class "B". Hence, S = 1.0, $\beta_0 = 2.5$, $T_B = 0.15$ and $T_C = 0.6$.

(ii) design ground acceleration in terms of g, $\alpha = 0.15$

Also, because the structure is of reinforced concrete, the critical viscous damping ζ is 5%. Hence, $\eta = (7/(2+\zeta))^{0.5} = 1$. The linear elastic response spectrum is reduced by the behaviour factor q in order to force the structure to resist the seismic actions in the inelastic range. This allows for the design based on lower seismic design forces..



$$\text{Spectral acceleration, } S_d(T_1) = \alpha.S.\eta.\frac{\beta_0}{q}, \qquad T_B \leq T_1 \leq T_C$$

$$= 0.15 \text{ x } 1.0 \text{ x } 1.0 \text{ x } \frac{2.5}{3.75}$$

$$= 0.1\text{g}$$

Base Shear

Base shear, $F_b = S_d(T_1).W = 0.1 \text{ x } (918 + 993 + 993) = 290.4$ kN

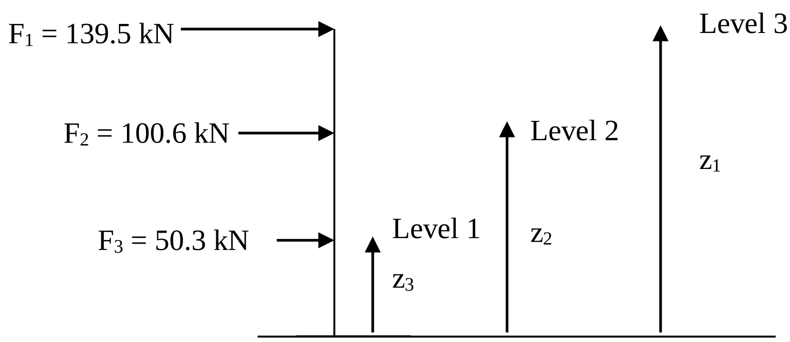
Seismic Forces Per Storey

Assuming that the base shear is linearly distributed along the height of the building, the seismic forces at level i are given by F_i where z_i is the height of the storey from the ground.

$$F_i = F_b.\frac{z_i.W_i}{\sum z_j.W_j}$$

$z_3 = 3$m, $z_2 = 6$m, $z_1 = 9$m
$\Sigma z_j W_j = 3 \text{ x } 993 + 6 \text{ x } 993 + 9 \text{ x } 918 = 17199$ kNm

Seismic force at Level 3, $F_1 = 290.4 \text{ x } (9 \text{ x } 918) / 17199 = 139.5$ kN
Seismic force at Level 2, $F_2 = 290.4 \text{ x } (6 \text{ x } 993) / 17199 = 100.6$ kN
Seismic force at Level 1, $F_3 = 290.4 \text{ x } (3 \text{ x } 993) / 17199 = 50.3$ kN



1.1.5.5.3 Seismic Loads using Multi-Modal Analysis

The flexibility matrix of the 3 storey shear frame model is obtained from the structural static analysis program by successively applying 1kN unit loads at each storey and each time measuring the total displacement at all the storey levels.

Flexibility Matrix

$$\underline{F} = \begin{pmatrix} f_{1,1} & f_{1,2} & f_{1,3} \\ f_{2,1} & f_{2,2} & f_{2,3} \\ f_{3,1} & f_{3,2} & f_{3,3} \end{pmatrix} = \begin{pmatrix} 0.040 & 0.024 & 0.009 \\ 0.024 & 0.022 & 0.009 \\ 0.009 & 0.009 & 0.007 \end{pmatrix} \quad \text{mm/kN}$$

The slab contribution to the flexural capacity of the frame has been neglected in that the effective width of the top flange of the beams is taken as the width of the web of the beam. Hence in reality, the slab would actually provide a slightly stiffer response to that predicted.

Stiffness Matrix

$$\underline{K} = \underline{F}^{-1} = \begin{pmatrix} 73.441 & -87.525 & 18.109 \\ -87.525 & 200.201 & -144.869 \\ 18.109 & -144.869 & 305.835 \end{pmatrix} \quad \text{kN/mm}$$

$$= \begin{pmatrix} 73441 & -87525 & 18109 \\ -87525 & 200201 & -144869 \\ 18109 & -144869 & 305835 \end{pmatrix} \quad \text{kN/m}$$

Mass Matrix

$$\underline{M} = \begin{pmatrix} 93.578 & 0 & 0 \\ 0 & 101.223 & 0 \\ 0 & 0 & 101.223 \end{pmatrix} \quad \text{tonnes}$$

The mass matrix is the seismic weight at each level as calculated includes the full dead load and 30% of the live load.

Eigenvalues ω^2

To find ω^2, we solve the eigenvalue equation $\left| \underline{K} - \omega^2 \underline{M} \right| = 0$

$$\underline{K} - \omega^2 \underline{M} = \begin{pmatrix} 73441 - 93.578\omega^2 & -87525 & 18109 \\ -87525 & 200201 - 101.223\omega^2 & -144869 \\ 18109 & -144869 & 305835 - 101.223\omega^2 \end{pmatrix}$$

Hence the eigenvalue equation is given by:

$\left| \underline{K} - \omega^2 \underline{M} \right| = 0.1006036 \times 10^{16} - 0.6718900 \times 10^{13}\,\omega^2 + 0.5545779 \times 10^{10}\,\omega^4 - 958809.146\,\omega^6 = 0$

$$\omega^2 = \begin{pmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \end{pmatrix} = \begin{pmatrix} 173.959 \\ 1449.824 \\ 4160.245 \end{pmatrix}$$

ω_I represents the fundamental natural circular frequency of this structure as it is the smallest. This gives the fundamental mode shape and the fundamental period.

Mode Shapes ϕ

The equation $[\,\underline{K} - \omega_i^2\,\underline{M}\,][\phi_i] = 0$ is solved for i = 1 to 3 to give the three different mode shapes.

$$\left[\underline{K} - \omega_i^{\,2}\,\underline{M}\right]\left[\phi_i\right] = [0] \qquad \text{for} \qquad i = 1, 2, 3$$

Normalising the first row to unity,

$$\underline{\phi} = \left[\underline{\phi_1} \quad \underline{\phi_2} \quad \underline{\phi_3}\right] = \begin{bmatrix} 1 & 1 & 1 \\ 0.715 & -0.906 & -2.840 \\ 0.297 & -0.939 & 3.726 \end{bmatrix}$$

ϕ_1 is the fundamental mode shape, i.e. the shape that requires the least amount of energy to vibrate.

Modal Mass (Generalized Mass), Participation Factor and Effective Mass

It is necessary to compute the contribution of each mode of vibration to the response of the structure. To this end the effective mass, which represents the structure as a single-degree-of-freedom system, is computed.
Modal mass (generalized mass) $GM^i = \phi^{iT}\,M\,\phi^i$, modal force amplitude $L^i = \phi^{iT}\,M\{1\}$, effective mass $m^i = L^{i\,2} / GM^i$ for i = 1 to 3

$$GM^1 = \phi^{1^T} M \phi^1 = \begin{pmatrix} 1 & 0.715 & 0.297 \end{pmatrix} \begin{pmatrix} 93.578 & 0 & 0 \\ 0 & 101.223 & 0 \\ 0 & 0 & 101.223 \end{pmatrix} \begin{pmatrix} 1 \\ 0.715 \\ 0.297 \end{pmatrix} = 154.215$$

$$L^1 = \phi^{1^T} M\{1\} = \begin{pmatrix} 1 & 0.715 & 0.297 \end{pmatrix} \begin{pmatrix} 93.578 & 0 & 0 \\ 0 & 101.223 & 0 \\ 0 & 0 & 101.223 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 195.959$$

$$m^1 = \frac{L^{1^2}}{GM^1} = \frac{195.959^2}{154.215} = 249.0$$

$$GM^2 = \phi^{2^T} M \phi^2 = \begin{pmatrix} 1 & -0.906 & -0.939 \end{pmatrix} \begin{pmatrix} 93.578 & 0 & 0 \\ 0 & 101.223 & 0 \\ 0 & 0 & 101.223 \end{pmatrix} \begin{pmatrix} 1 \\ -0.906 \\ -0.939 \end{pmatrix} = 265.846$$

$$L^2 = \phi^{2^T} M\{1\} = \begin{pmatrix} 1 & -0.906 & -0.939 \end{pmatrix} \begin{pmatrix} 93.578 & 0 & 0 \\ 0 & 101.223 & 0 \\ 0 & 0 & 101.223 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \text{-}93.141$$

$$m^2 = \frac{L^{2^2}}{GM^2} = \frac{-93.141^2}{265.846} = 32.6$$

$$GM^3 = \phi^{3^T} M \phi^3 = \begin{pmatrix} 1 & -2.840 & 3.726 \end{pmatrix} \begin{pmatrix} 93.578 & 0 & 0 \\ 0 & 101.223 & 0 \\ 0 & 0 & 101.223 \end{pmatrix} \begin{pmatrix} 1 \\ -2.840 \\ 3.726 \end{pmatrix} = 2315.248$$

$$L^3 = \phi^{3^T} M\{1\} = \begin{pmatrix} 1 & -2.840 & 3.726 \end{pmatrix} \begin{pmatrix} 93.578 & 0 & 0 \\ 0 & 101.223 & 0 \\ 0 & 0 & 101.223 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 183.271$$

$$m^3 = \frac{L^{3^2}}{GM^3} = \frac{183.271^2}{2315.248} = 14.5$$

Hence,

$$\begin{aligned} GM &= \begin{bmatrix} 154.215 & 265.846 & 2315.248 \end{bmatrix} \\ L &= \begin{bmatrix} 195.959 & -93.141 & 183.271 \end{bmatrix} \\ m &= \begin{bmatrix} 249 & 32.6 & 14.5 \end{bmatrix} \\ &= \begin{bmatrix} 84\% & 11\% & 5\% \end{bmatrix} \end{aligned}$$

The effective mass associated with the fundamental mode of vibration, m_1 is clearly the largest at 84% of the total mass, and hence the forces imparted upon the structure vibrating in this fundamental mode are the most damaging (i.e. the critical case).

Natural Periods of Vibration

$$T = \frac{2\pi}{\omega} = \begin{pmatrix} T_1 \\ T_2 \\ T_3 \end{pmatrix} = \begin{pmatrix} 0.48 \\ 0.17 \\ 0.10 \end{pmatrix} \text{seconds}$$

The fundamental period is 0.48 seconds.

Inelastic Response Spectrum of EC8

From previous soil response analysis of **Section 1.1.2.15.1**,

(i) Subsoil class “B”. Hence, S = 1.0, $\beta_0 = 2.5$, $T_B = 0.15$ and $T_C = 0.6$.

(ii) Design ground acceleration in terms of g, $\alpha = 0.15$

Also, because the structure is of reinforced concrete, the critical viscous damping ζ is 5%. Hence, $\eta = (7/(2+\zeta))^{0.5} = 1$. The behaviour factor q = 3.75, as before.

$$\begin{aligned} \text{Spectral acceleration, } S_d(T_1 = 0.48) &= \alpha.S.\eta.\frac{\beta_0}{q} \qquad T_B \le T_1 \le T_C \\ &= 0.15 \text{ x } 1.0 \text{ x } 1.0 \text{ x } \frac{2.5}{3.75} \\ &= 0.100g \end{aligned}$$

$$\begin{aligned} \text{Spectral acceleration, } S_d(T_2 = 0.17) &= \alpha.S.\eta.\frac{\beta_0}{q} \qquad T_B \le T_2 \le T_C \\ &= 0.15 \text{ x } 1.0 \text{ x } 1.0 \text{ x } \frac{2.5}{3.75} \\ &= 0.100g \end{aligned}$$

$$\text{Spectral acceleration, } S_d(T_3 = 0.10) = \alpha.S.\left[1 + \frac{T_3}{T_B}.(\eta.\frac{\beta_0}{q} - 1)\right], \qquad 0 \le T_3 \le T_B$$

$$= 0.15 \text{ x } 1.0 \text{ x } \left[1 + \frac{0.10}{0.15}.(1 \text{ x } \frac{2.5}{3.75} - 1)\right]$$

$$= 0.117\text{g}$$

Modal Force Response

The modal force responses are the horizontal loads at each level for the different mode shapes.

$$f_i = \underline{M}\phi^i \frac{L_i}{GM_i} S_{e,i} \qquad \text{for } i = 1, 2, 3$$

$$f_1 = \underline{M}\underline{\phi}^1 \frac{L_1}{GM_1} S_{d,1} = \begin{pmatrix} 93.578 & 0 & 0 \\ 0 & 101.223 & 0 \\ 0 & 0 & 101.223 \end{pmatrix} \begin{pmatrix} 1 \\ 0.715 \\ 0.297 \end{pmatrix} \frac{195.959}{154.215} 0.100\text{x}9.81 = \begin{pmatrix} 116.6 \\ 90.2 \\ 37.4 \end{pmatrix} \text{ kN}$$

$$f_2 = \underline{M}\underline{\phi}^2 \frac{L_2}{GM_2} S_{d,2} = \begin{pmatrix} 93.578 & 0 & 0 \\ 0 & 101.223 & 0 \\ 0 & 0 & 101.223 \end{pmatrix} \begin{pmatrix} 1 \\ -0.906 \\ -0.939 \end{pmatrix} \frac{-93.141}{265.846} 0.100\text{x}9.81 = \begin{pmatrix} -32.2 \\ 31.5 \\ 32.7 \end{pmatrix} \text{ kN}$$

$$f_3 = \underline{M}\underline{\phi}^3 \frac{L_3}{GM_3} S_{d,3} = \begin{pmatrix} 93.578 & 0 & 0 \\ 0 & 101.223 & 0 \\ 0 & 0 & 101.223 \end{pmatrix} \begin{pmatrix} 1 \\ -2.840 \\ 3.726 \end{pmatrix} \frac{183.271}{2315.248} 0.117\text{x}9.81 = \begin{pmatrix} 8.5 \\ -26.1 \\ 34.3 \end{pmatrix} \text{ kN}$$

$$\underline{f} = \begin{pmatrix} 116.6 & -32.2 & 8.5 \\ 90.2 & 31.5 & -26.1 \\ 37.4 & 32.7 & 34.3 \end{pmatrix} \text{ kN}$$

Modal Combination using SRSS

As the natural periods of vibration are not too closely spaced between the different modes, the Square Root of the Sum of Squares, SRSS, is used to combine the modal responses providing a reasonable estimate of the total maximum response.

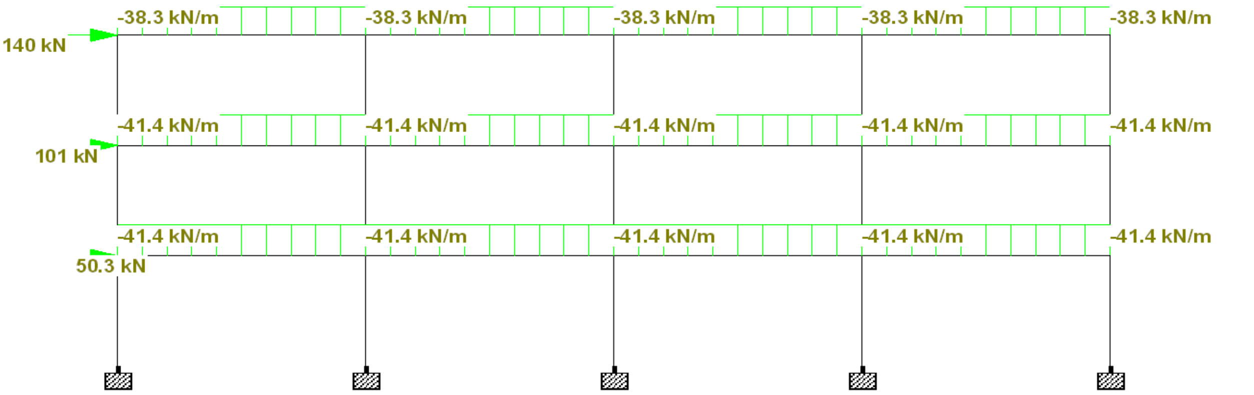
$$\underline{ff}_j = \sqrt{\sum_i f_{j,i}^2} = \begin{pmatrix} 121.3 \\ 99.0 \\ 60.4 \end{pmatrix} \text{ kN}$$

$$\Delta = \frac{|F - ff|}{F} \text{x}100\% = \begin{pmatrix} 13.0 \\ 1.6 \\ 20.1 \end{pmatrix} \%$$

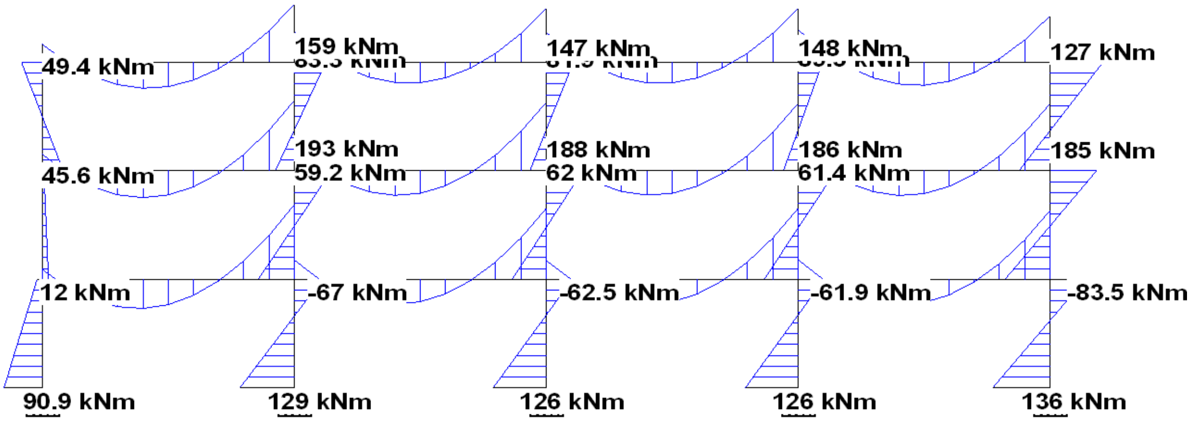
Since this structure is regular, higher modes of vibrations are rather insignificant. The fundamental period is also less than 2, hence the simplified equivalent lateral force analysis method is sufficient.

1.1.5.5.4 Structural Static Analysis

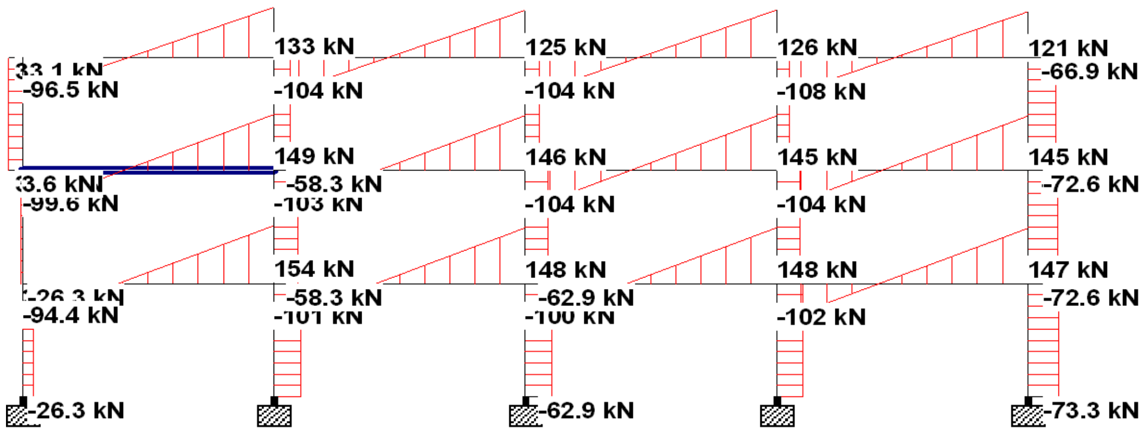
A structural static analysis is now carried out using the forces computed with the simplified equivalent lateral force and the vertical loads, given by 1.0 G_k + 0.3 Q_k, shown in below. The next two figures show the bending moment and the shear force diagrams.



Loading



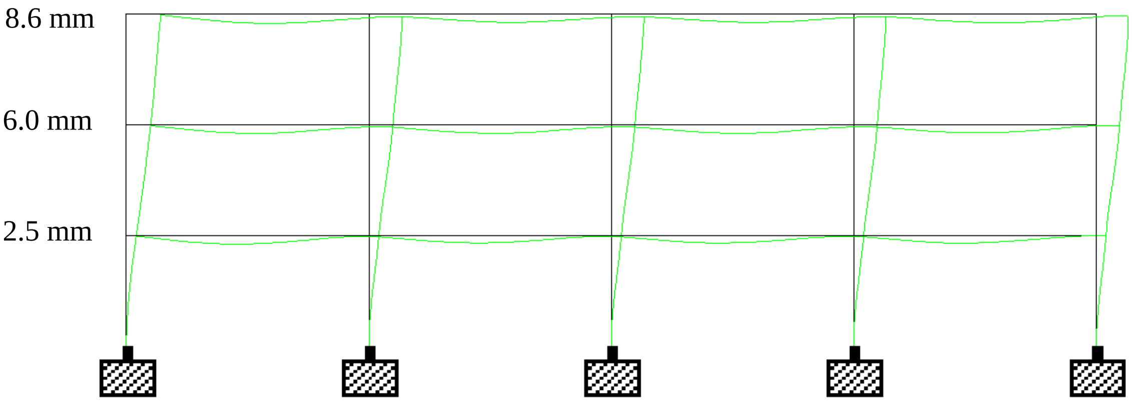
Bending Moment Diagram



Shear Force Diagram

1.1.5.5.5 Serviceability Limit Verification

The displacements computed in the analysis, shown below, are used to verify that the serviceability limit has not been exceeded.



Horizontal Displacements

EC8 sets limits on the deformation that is allowed at each level under seismic action. This relative displacement (drift) should not exceed 0.4% of the height of the story, thus for each storey of the structure this drift should not exceed 12mm.

The figure shows the displacements at each level:
Storey 3: Δ_r = 0.0086-0.0060 = 0.0026m (< 0.012m)
Storey 2: Δ_r = 0.0060-0.0025 = 0.0035m (< 0.012m)
Storey 1: Δ_r = 0.0025-0.0000 = 0.0025m (< 0.012m)

What we have here is the elastic displacement due to the inelastic forces, however this is unacceptable for design purposes. To obtain the inelastic displacements these elastic displacements are multiplied by the behaviour factor, q. This gives:

Storey 3: Δ_r = 0.0026 x 3.75 = 0.00975m (< 0.012m)
Storey 2: Δ_r = 0.0035 x 3.75 = 0.01313m (> 0.012m)
Storey 1: Δ_r = 0.0025 x 3.75 = 0.00938m (< 0.012m)

Hence on the second storey the drift fractionally exceeds the limit, however the analysis does not take into account the stiffness of the slab. Therefore this design is probably satisfactory but may entail slight adjustments to the section in question.

1.1.5.6 MSC.NASTRAN Decks

There are two parts to a response spectrum analysis, namely the generation of the spectrum and the use of the spectrum to compute the dynamic response. The response spectrum analysis is the only dynamic analysis method that requires a SUPORT entry to define the input DOFs for the spectra.

The procedure involves two stages. First the applied loads or base excitations are converted in a direct transient solution (SOL 109) into a spectrum table consisting of peak response magnitudes for a set of single degree-of-freedom oscillators. Each oscillator is a scalar spring/mass/damper having a different natural frequency and damping ratio. This stage is optional since the shock spectrum data is frequently given in the contractual design specifications or, in the case of earthquakes, is available through governmental agencies. The second stage of the analysis consists of a modal analysis of the structure, data recovery, and the response calculation that combines the modal properties of the analysis model with the spectrum data of the applied loads. This stage is performed in a modal analysis solution sequence (SOL 103). If a database was saved from the first stage, a restart will provide the spectrum data automatically. Otherwise, the spectrum data must be supplied in a direct tabular input (response versus natural frequency for several damping ratios).

1.1.5.6.1 Theoretical Background

Starting with a modal transient analysis, the general approximation for a response quantity, u_k, is

$$u_k(t) = \sum_i \phi_{ik}\xi_i(t) \qquad \text{Eq. 6-28}$$

where ϕ and ξ are the modal outputs and generalized displacements. The actual modal equations are

$$\ddot{\xi}_i + g_i\omega_i\dot{\xi}_i + \omega_i^2\xi_i = \lfloor \phi_i \rfloor^T \{P(t)\} \qquad \text{Eq. 6-29}$$

where P is the vector of loading functions. For loading due to base accelerations, the equivalent inertial loads are

$$\{P(t)\} = -[M_{aa}][D_{ar}]\{\ddot{u}_r(t)\} \qquad \text{Eq. 6-30}$$

where the columns of $[D_{ar}]$ represent vectors of rigid body motions of the whole structure and the accelerations correspond to the base motions. Substituting Eq. 6-30 into Eq. 6-29 and combining terms we can separate the modal quantities from the transient solutions. First we will develop the transient response functions. We begin by calculating the responses

$$\ddot{x}_r + g\omega\dot{x}_r + \omega^2 x_r = \ddot{u}_r(t) \qquad \text{Eq. 6-31}$$

where x_r is a response function in direction r, and is a function of the variables ω, g, and t. The peak values of x_r, obtained over a range of frequencies and damping factors is called the response spectrum for the excitation, $\ddot{u}_r$.

Next, from the normal mode analysis, we define the participation factors ψ_{ir}, for mode i and direction r, as

$$\psi_{ir} = -\lfloor \phi_i \rfloor^T [M_{aa}]\{D_{ar}\} \qquad \text{Eq. 6-32}$$

Then, from Eq. 6-28, the actual transient response at a physical point is

$$u_k(t) = \sum_i \sum_r \phi_{ik}\psi_{ir}x_r(\omega_i, g_i, t) \qquad \text{Eq. 6-33}$$

The peak magnitudes of u_k in Eq. 6-33 are usually dominated by the peak values of $x(t)$ occurring at the natural frequencies. In spectrum analysis the peak values of u_k are approximated by combining functions of the peak values, $\bar{x}_{ri}(\omega_i,g_i) = \max|x_{ri}(\omega_i,g_i,t)|$, in the approximation

$$\bar{u}_k(t) \cong \sum_i \sum_r |\phi_{ik}| \, |\psi_{ir}\bar{x}_{ri}(\omega_i,g_i)| \qquad \text{Eq. 6-34}$$

ABS Option. Eq. 6-33 and Eq. 6-34 define the ABS (Absolute Value) option. This method assumes the worst case scenario in which all of the modal peak values for every point on the structure are assumed to occur at the same time and in the same phase. Clearly in the case of a sudden impact, this is not very probable because only a few cycles of each mode will occur. However, in the case of a long term vibration, such as an earthquake when the peaks occur many times and the phase differences are arbitrary, this method is acceptable.

A second way of viewing the problem is to assume that the modal magnitudes and phases will combine in a probabalistic fashion. If the input loads are behaving randomly, the probable (RMS) peak values are

$$\bar{u}_k \cong \sqrt{\sum_i (\phi_{ik}\bar{\xi}_i)^2} \qquad \text{Eq. 6-35}$$

where the average peak modal magnitude, $\bar{\xi}_i$ is

$$\bar{\xi}_i = \sqrt{\sum_r \left(\psi_{ir}\bar{x}_r(\omega_i,g_{i_i})\right)^2} \qquad \text{Eq. 6-36}$$

SRSS Method. This approach is known as the SRSS (square root of sum-squared) method. Note that the results in each direction are summed in vector fashion for each mode first, followed by an SRSS calculation for all modes at each selected output quantity u_k. It is assumed that the modal responses are uncorrelated and the peak value for each mode will occur at a different time. These results are optimistic and represent a lower bound on the dynamic peak values.

The SRSS method may underestimate the actual peaks since the result is actually a probable peak value for the period of time used in the spectrum analysis. The method is normally augmented with a safety factor of 1.5 to 2.0 on the critical outputs.

NRL Method. As a compromise between the two methods above, the NRL (Naval Research Laboratories) method was developed. Here, the peak response is calculated from the equation

$$\bar{u}_k \cong |\phi_{jk}\bar{\xi}_j| + \sqrt{\sum_{i \neq j} (\phi_{ik}\bar{\xi}_i)^2} \qquad \text{Eq. 6-37}$$

where the j-th mode is the mode that produces the largest magnitude in the product $\phi_{jk}\bar{\xi}_j$. The peak modal magnitudes, $|\phi_{jk}\bar{\xi}_j|$, are calculated with Eq. 6-36.

The rationale for the method is that the peak response will be dominated by one mode and the SRSS average for the remaining modes could be added directly. The results will fall somewhere between the ABS and SRSS methods.

Modes that are close in frequency may have their peak response occur at about the same time (and with the same phase). For this reason, the SRSS and NRL methods contain a provision to sum modal responses via the ABS method for modes that have closely spaced natural frequencies. Close natural frequencies are defined by frequencies that meet the following inequality:

$$f_{i+1} < \text{CLOSE} \cdot f_i$$

The value for CLOSE is set by PARAM,CLOSE (the default is 1.0).

The modal summation option is set via PARAM,OPTION (the ABS method is the default). Both PARAM,OPTION and PARAM,CLOSE may be set in any subcase, allowing summation by several conventions in a single run.

1.1.5.6.2 Response Spectra Generation

Response spectra are generated in the transient response solution sequences (SOL 109 for direct and SOL 112 for modal). Transient response input is required to apply the transient excitation to the base structure. Additional input is described.

Case Control Command	Description
XYPLOT SPECTRAL	Compute spectra.
XYPUNCH SPECTRAL	Punch spectra for subsequent use.

```
$  Plot absolute acceleration spectra for grid point 85, T3 component
XYPLOT     ACCE     SPECTRAL     1      /85(T3RM)

$  Punch relative displacement spectra for grid point 3, T1 component
XYPUNCH    DISP     SPECTRAL     1      /3(T1IP)
```

Relative and absolute spectra are denoted by IP and RM, respectively, in the parentheses of the curve request.

Bulk Data Entry	Description
PARAM,RSPECTRA,0	Requests calculation of spectra.
DTI, SPSEL, 0	Header for DTI.
DTI, SPSEL, 1	Selects oscillator frequencies, oscillator damping values, and grid points at which spectra will be computed.
FREQi	Specifies oscillator damping values.
FREQi	Specifies oscillator frequencies.

There are two FREQi entries: one to specify oscillator frequencies (i.e., frequencies for which spectra will be computed) and the other to specify oscillator damping. (Note that damping for the base structure is specified in another manner, such as with the TABDMP1 entry used for modal transient response analysis.)

1.1.5.6.3 Peak Response Calculation

Response spectrum application is a postprocessing function of normal modes analysis (SOL 103). It is run in the normal modes solution sequences, so the modelling and analysis considerations that apply for normal modes analysis also apply for response spectrum application. The additional considerations also need to be followed:

1. The structure is run as an unrestrained model in the direction(s) of the load spectrum application.
2. A large mass, on the order of 10^3 to 10^6 times the mass of the structure, must be used at the structure's base grid points where the input occurs. One way to do this is to use RBEs or MPCs to connect the base points to a separate grid point, and apply the large mass to that separate grid point. This separate grid point is where the spectrum is applied.
3. A SUPORT Bulk Data entry is required at the spectrum input location.
4. The modes must be mass normalized (which is the default).

The spectra that MSC.Nastran can apply are absolute acceleration, relative displacement, and relative velocity spectra. You specify A, D, or V for acceleration displacement or velocity, respectively, to specify the spectrum type. Use all modes within the frequency range specified by the spectrum, but do not use modes outside of the spectrum range. Usually, spectra to apply are considered to have zero values outside of their range of definition; for example, an absolute acceleration spectrum defined from 0 to 30 Hz is assumed to be zero beyond 30Hz. However, MSC.Nastran extrapolates spectral values for modes beyond the spectral range, which may lead to unexpected answers. You can limit the number of modes used in the spectrum application by limiting the number of computed modes (via the EIGRL or EIGR entry) or by using PARAM,HFREQ,r (where r is the highest frequency mode to use) or PARAM,LMODES,n (where n is the number of lowest modes to use).

Consider the entire response spectrum process—generation and application—as a two-step process. Step 1 is generation of the response spectra and Step 2 is the application of the response spectra. For a given input, transient applied to the base structure (Step 1), the same stresses occur (Step 2) regardless of whether acceleration or displacement spectra were computed in Step 1. However, displacements and accelerations are different, because answers computed by using the absolute acceleration spectrum contain the rigid body contribution, whereas answers computed by using the relative displacement spectrum do not contain the rigid body contribution. Displacement and acceleration responses can be made equal regardless of which spectra was used by using PARAM,LFREQ,0.01 (or some other small number) to remove the rigid body mode contribution from the answers. Stresses and other element quantities are unaffected by the contribution of any rigid body modes. The same situation applies to relative velocity spectra as to relative displacement spectra.

Case Control Command	Description
METHOD	Selects eigenvalue extraction method.
SDAMP	Selects the TABDMP1 Bulk Data entry.
DLOAD	Selects the DLOAD Bulk Data entry.
Bulk Data Entry	**Description**
PARAM,SCRSPEC,0	Requests response spectrum application.
EIGR or EIGRL	Eigenvalue extraction method.
TABDMP1	Specifies damping for the structure.
DLOAD	Defines spectrum multipliers.
DTI,SPECSEL,0	Header for DTI.
DTI,SPECSEL,1	Specifies type of spectrum (A, V, or D) and selects damping. A = absolute acceleration spectrum. V = relative velocity spectrum. D = relative displacement spectrum.
TABLED1	Specifies input spectrum values.
SUPORT	Specifies input spectrum grid points.
CONM2,CMASS2, etc.	Defines large mass used for the input spectrum.
PARAM,OPTION,a	Specifies modal combination method (a = ABS [default], SRSS, or NRL).
PARAM,CLOSE,r	Specifies closeness parameter for modal combinations (the default is 1.0).

All input listed in the table is required with the exception of PARAM,OPTION and PARAM,CLOSE.

The time step (field 4 on the TSTEP Bulk Data entry, DT) should not be changed during the run, because MSC.Nastran uses only the initial DT specification for the entire response spectrum generation run and therefore wrong answers could occur. The time step, DT, and time duration, (where is the number of time increments), must take into account the loading, the base structure, and the frequency range of the spectra generation. The time step must take into account the frequency content of the applied excitation, the frequencies of the base structure, and the highest frequency for which spectra are to be generated. There must be enough time steps per cycle of response for both the base structure and the highest frequency oscillator in order to accurately predict the peak response; 5 to 10 steps per cycle represent a typical value. In addition, the time duration of the loading, the frequencies of the base structure, and the lowest oscillator frequency must be considered when defining the time duration. There must be a long enough time duration of response both for the base structure and the lowest frequency oscillator in order to accurately predict the peak response. For short duration loadings, the peak response often occurs well after the load has peaked. Initial conditions (specified via the TIC Bulk Data entry) are not used in response spectrum generation. Initial conditions are used in the calculation of the transient response of the base structure, but the calculation of the peak oscillator responses (i.e., the response spectrum calculation) ignores any initial conditions.

1.1.6 Reinforced Concrete Design to EC8 for Earthquake Effects Based on Response Spectrum Analysis

1.1.6.1 Concepts of Ductility

The definition of ductility is

$$\mu = \frac{\text{Deformation Quantity at Ultimate}}{\text{Deformation Quantity at Yield}}$$

The deformation considered may be curvature μ_ϕ, rotation μ_θ, or displacement μ_δ, commonly the latter.

1.1.6.1.1 Section Ductility

Curvature ductility, $\mu_\phi = \phi_u / \phi_y$

where ϕ_y = yield of reinforcement
- higher A_s, A_{sv}, and f_y would increase ductility as ϕ_y decreases
- A_{sv} increases confinement hence increasing ductility

where ϕ_u = ultimate compressive strain of concrete, i.e. concrete crushing
- higher axial load increases compressive strain, hence decreasing ductility as ϕ_u decreases

1.1.6.1.2 Member Ductility

Displacement ductility, $\mu_\Delta = \dfrac{\Delta_u}{\Delta_y}$ and Rotation ductility, $\mu_\theta = \dfrac{\theta_u}{\theta_y}$

where Δ_y = yield of reinforcement
- higher A_s, A_{sv}, and f_y would increase ductility as Δ_y decreases

where Δ_u = ultimate compressive strain of concrete, i.e. concrete crushing or buckling or fracture of rebars
- higher axial load increases P-Δ effects, hence decreasing ductility as Δ_u decreases
- higher levels of A_{sv} will decrease risk of buckling of rebars, hence increasing ductility by increasing Δ_u
- strain hardening rebars will promote spread of inelasticity, hence lengthening the plastic hinge length, increasing ductility as Δ_u increases

1.1.6.1.3 Structural Ductility

Structural ductility = Total drift at top of structure / Total base shear
or Structural ductility = behaviour factor q = ability to achieve inelastic deformations

I. Capacity design concepts should be employed. The structure is viewed comprising of dissipative and non-dissipative parts. The dissipative zone mobilizes desired failure mode and avoids collapse and maximizes the energy dissipation.

(a) The dissipative zones are dimensioned first and carefully detailed to posses maximum ductility

(b) Their likely sources of over-strength is estimated:-
- higher concrete compressive strength, f_{cu}
- higher yield strength of steel, f_y
- larger area of steel rebars, A_s
- slab contribution to effective flange compressive area in the case of beams
- confinement
- strain-hardening characteristics of steel

(c) The non-dissipative parts are then designed to withstand forces consistent with the strength of dissipative parts, including over-strength. Hence the non-dissipative parts do not reach M_P i.e. its plastic capacity until the dissipative parts have reached M_P and absorbed considerable energy through ductility.

II. Continuity and redundancy between members to ensure clear load path

III. Regularity of mass, stiffness and strength distribution to avoid torsional effects and soft-storeys

IV. Reduced mass

V. Sufficient stiffness to avoid highly flexible structures with significant P-Δ effects and non-structural damage.

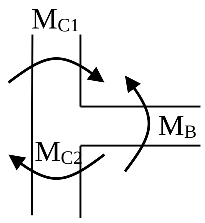
1.1.6.2 Capacity Design for Optimum Location and Sequence of Attainment of Member Capacity

Whereas in static design, under-estimating the member strength is an added safety procedure, this is not the case in seismic design. This is because the location and sequence of attainment of member capacity are very important for the collapse prevention and optimum response, respectively.

Location of attainment of M_P → Hinges should form in beams and not columns in order to avoid an unstable mechanism that will lead to collapse.

Sequence of attainment of M_P → Using the ductility of elements to absorb energy instead of the yield strength is a more optimum method.

We thus require beam hinges and not column hinges. Except at the base of the column, hinges are not accepted anywhere else on the column. The method of utilizing capacity design is as follows



Obtain the structural member forces from structural analysis of the earthquake excitation

I. Design beam to resist M_B

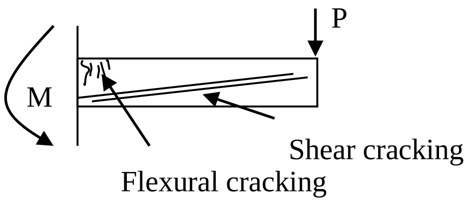
II. Estimate the over-strength of the beam, the sources of which are f_c, f_y, A_s, slab contribution; Say $M_{Bactual} = 1.25M_B$

III. Scale up M_{C1} and M_{C2} to satisfy equilibrium with $1.25M_B$. If the column stiffness were equal, then $M_{C1}=M_{C2}=0.625M_B$ instead of $0.5M_B$.

IV. Hence, the beam plastic hinge will definitely occur first, and the energy will then be absorbed by the ductility of the member. The plastic capacity of the columns won't be reached; instead the mechanism collapses when the ductility limit of the beam has been attained.

1.1.6.3 Capacity Design for Favourable Mechanism of Deformation

Flexural deformation is the more favourable mechanism of dissipation of energy than shear deformation because it is more ductile. To ensure flexural deformation precedes shear deformation, consider the design for a cantilever.



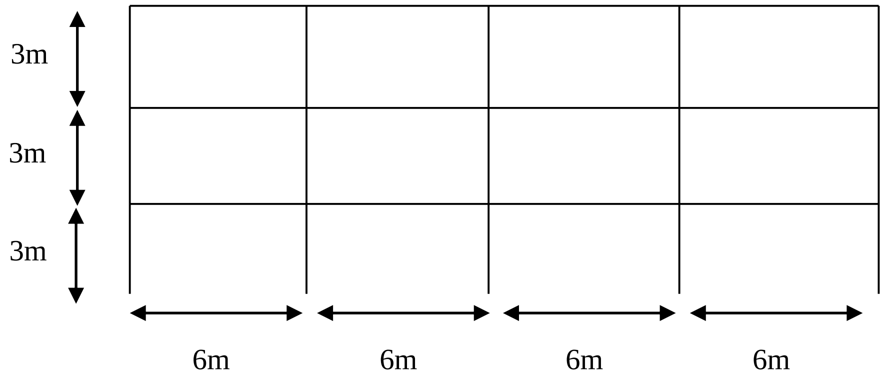
1. Design M=PL, hence design beam for bending capacity M=PL.
2. Estimate over-strength of $M_{actualcapacity} = 1.25M = 1.25PL$.
3. Design beam for shear force in equilibrium with 1.25PL, i.e. $V_{design} = 1.25PL$.

A point to note is that having strain hardening steel rebars is favourable for an efficient bending deformation energy dissipating mechanism. Perfectly plastic rebars will cause the formation of only one flexural crack. Hence the plastic hinge length will be short and hence the ductility requirement at that section is considerable. Strain hardening rebars will cause the formation of many flexural cracks. Hence the plastic hinge length will be long and hence the ductility requirement per section along the hinge length is reduced. The latter represents a far more efficient energy dissipating mechanism.

In the case of RC walls, the shear force consistent with the flexural strength, increased by a factor of about 20% in recognition of i) unintentional increase in steel yield f_y, ii) round up of A_s, iii) strain hardening of flexural reinforcement and iv) additional f_{cu}, is used to determine the shear capacity needed. Hence, flexural failure precedes shear failure.

1.1.6.4 Introduction of Example Problem

Following the determination of the seismic action, the reinforced concrete beams and columns are now designed. The design of the beams is based on EC2, which governs the design of concrete structures, and that of the columns is based on EC8, which makes provisions for the earthquake resistance of structures. The regularity of the structure in plan enables the analysis to be based on a typical frame as shown below. The floor slabs are assumed to be infinitely rigid (i.e. no large openings etc.) in order to distribute the floor shear forces amongst the columns in diaphragm action.



1.1.6.5 Design Data

Material used for design: Concrete class C25/30, longitudinal steel S400, transverse steel S220
Concrete cover to reinforcement is 30mm.

$\Rightarrow$ Concrete design strength, f_{cd} = 25/1.5 = 16.7MPa
$\Rightarrow$ Longitudinal steel strength, f_{yd} = 400/1.15 = 347.8MPa
$\Rightarrow$ Transverse steel strength, f_{yd} = 220/1.15 = 191.3MPa

1.1.6.6 Beam Design for Vertical Loading According to EC2

1.1.6.6.1 Structural Loading

Assume a concrete volumetric weight of 25 kN/m^3.
Initial sizing of beam, web = 0.3m, depth = 0.5m from previous project.

Loads on Slabs

Self-weight of slab + partitions, G = 6 kN/m^2
Live load, Q = 3 kN/m^2

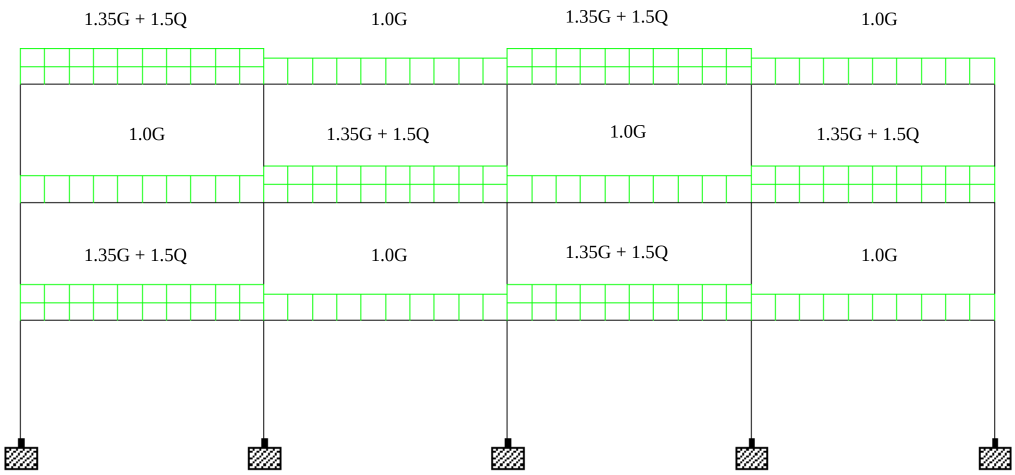
Loads on Beams

Dead load on beam, G_k = self-weight + slab dead load x width = 0.3 x 0.5 x 25 + 6 x 5 = 33.75 kN/m
Live load on beam, Q_k = slab live load x width = 3 x 5 = 15 kN/m

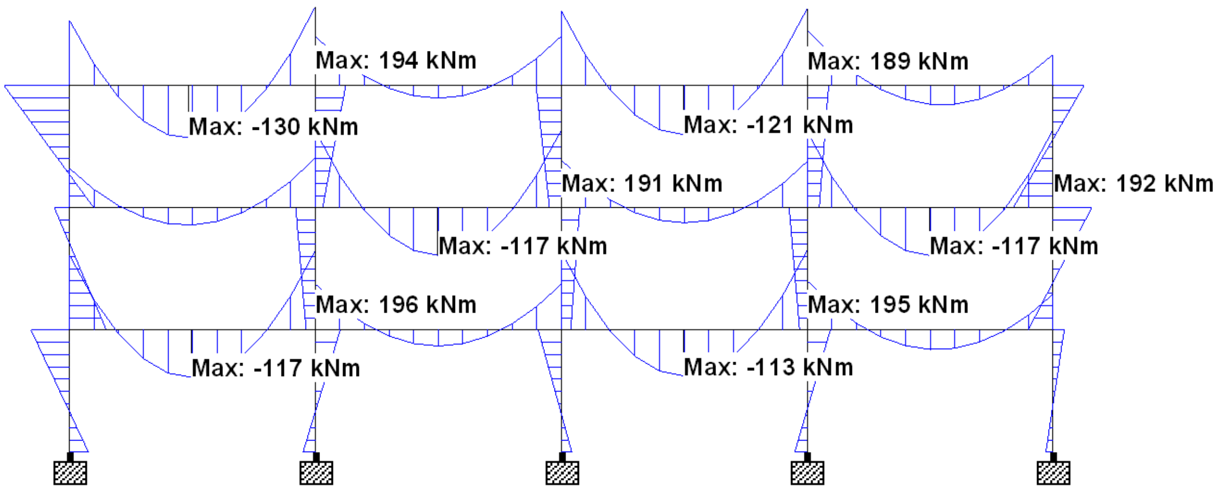
1.1.6.6.2 Determination of Maximum Action-Effects

The maximum bending moment is calculated for the different combinations of 1.35G + 1.5Q. The following gives the maximum elastic bending moment in the spans and the maximum elastic bending moment in the supports. The range of values for both the moments in the spans and the supports is not very large so only one beam needs to be designed for the entire structure.

MAXIMUM BENDING MOMENT IN SPANS WITH 1.35G+1.5Q

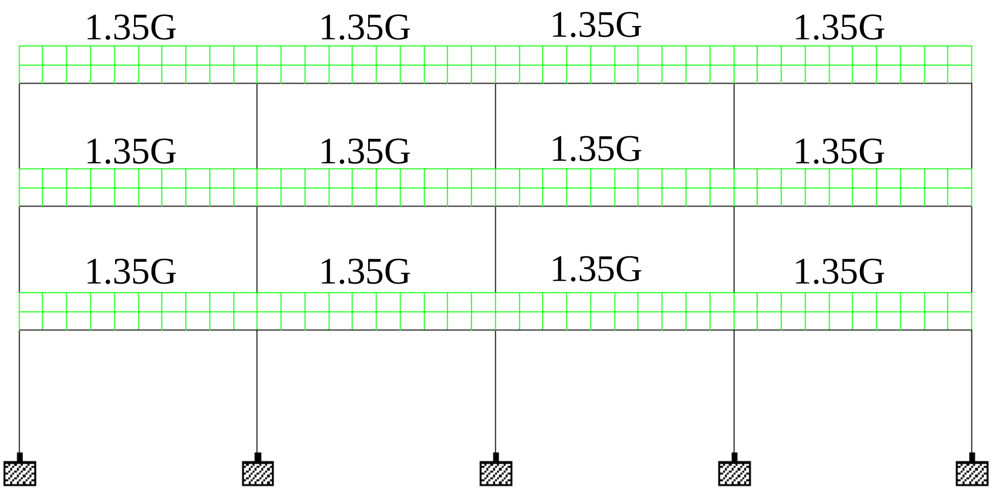


LOADING DIAGRAM



BENDING MOMENT DIAGRAM

MAXIMUM BENDING MOMENT IN SUPPORT



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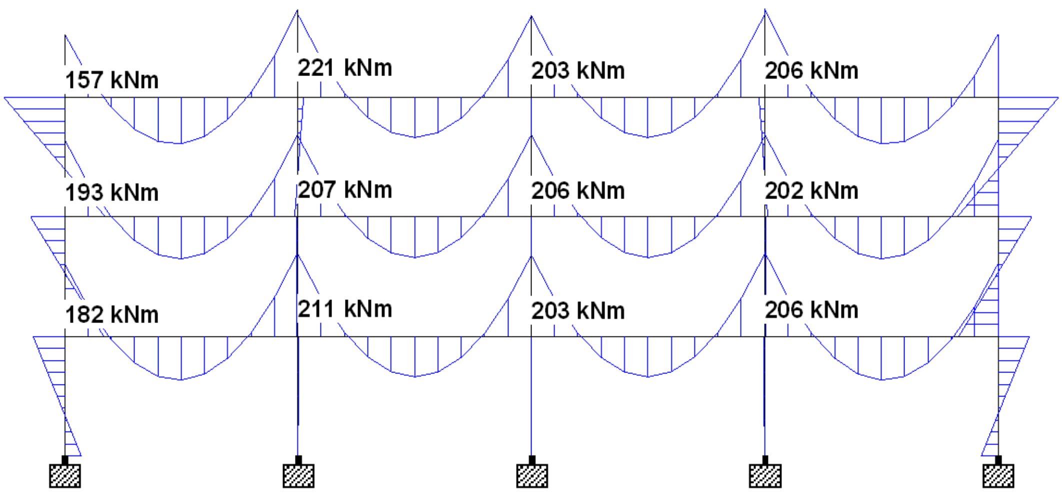
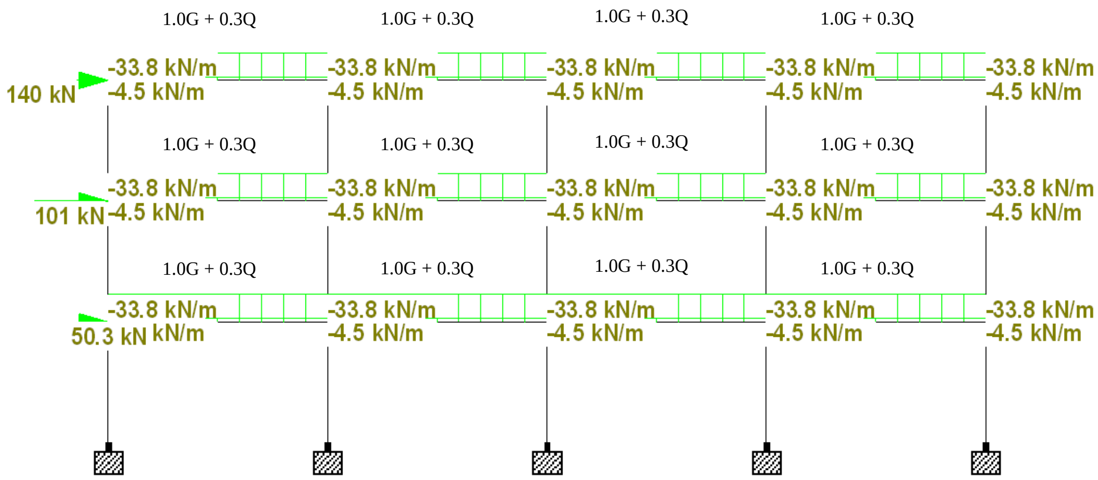
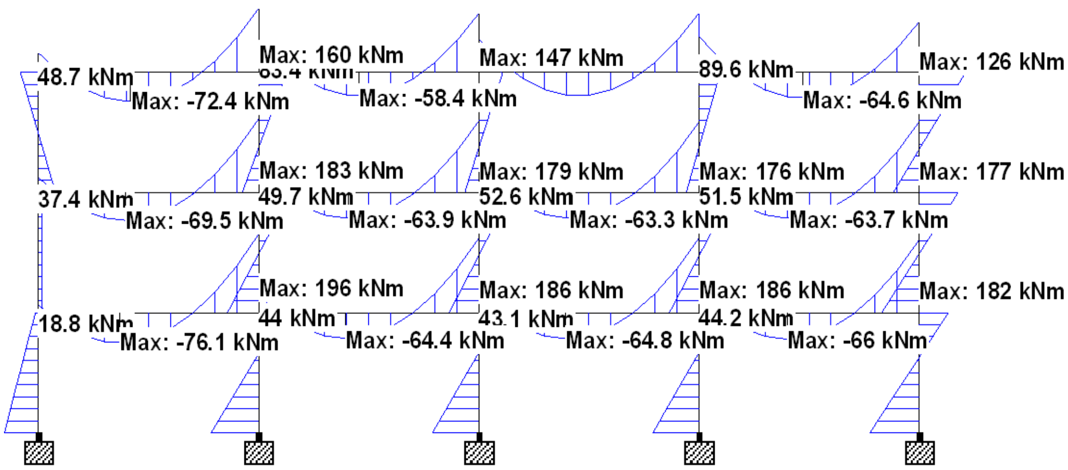


BENDING MOMENT DIAGRAM

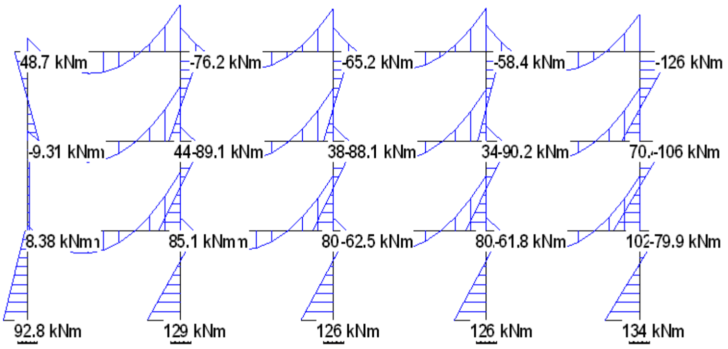
SEISMIC LOADCASE G+0.3Q+E



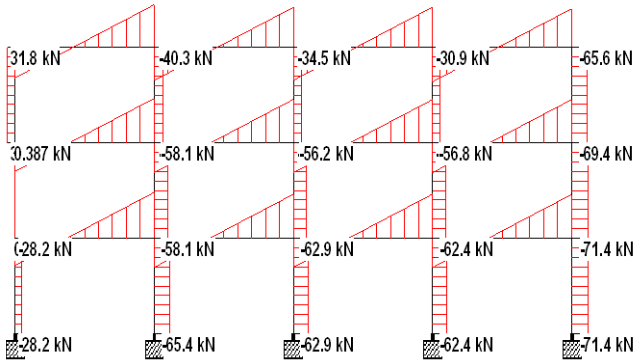
LOADING DIAGRAM



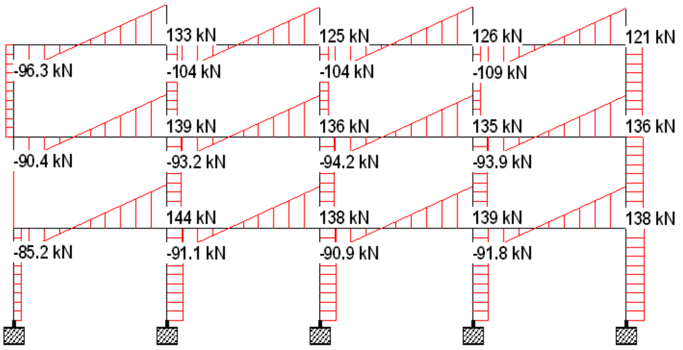
BENDING MOMENT DIAGRAM FOR BEAMS

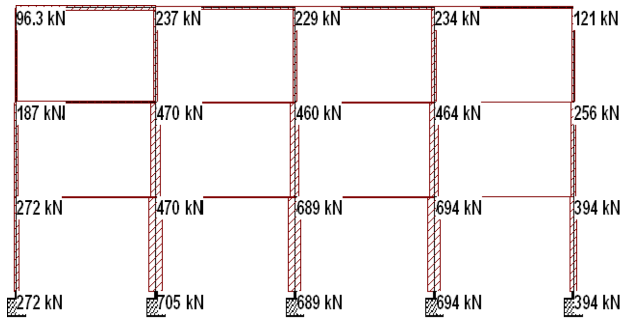


BENDING MOMENT DIAGRAM FOR COLUMNS



SHEAR FORCE DIAGRAM FOR COLUMNS





AXIAL FORCE DIAGRAM FOR COLUMNS

1.1.6.6.3 Design for Bending

Redistribution

Redistribution is carried out in order to optimise the design by obtaining a more uniform distribution of the action effects. The maximum value of the bending moment is 221 kNm. The redistribution is to be carried out for each beam.
First a 15% redistribution is applied, i.e. $\delta = 0.85$:

$M_{Sd} = 0.85 \times 221 = 187.85$ kNm

First the normalised moment, μ, is calculated: $\mu = \dfrac{M_{Sd}}{bd^2 f_{cd}}$

$$\Rightarrow \qquad \mu = \frac{187.85}{0.3 \times 0.47^2 \times 16.7 \times 10^3} = 0.17 \Rightarrow \text{Table 1} \Rightarrow \frac{x}{d} = 0.28$$

The condition stipulated in EC2 is that $\delta \geq 0.44 + 1.25\dfrac{x}{d}$. This is to ensure that the beam is adequately ductile. Since 0.44+1.25x0.28 = 0.79, is less than 0.85, so the design can proceed, taking maximum support moments of 187.85 kNm.

To find the maximum moments in the midspan after redistribution:

$$M^+_{max} = M_L + \frac{\left(\dfrac{M_R - M_L}{\ell} + \dfrac{p\ell}{2}\right)^2}{2p} = -160 + \frac{\left(\dfrac{-187.85 + 160}{6} + \dfrac{68.1 \times 6}{2}\right)^2}{2 \times 68.1} = 132.7\text{kNm}$$

New values of the maximum shear force can be checked by taking moments about the support. In general this does not make a difference, e.g. the maximum value of 215kN was reduced to 210kN. These changes are negligible.

Design of Longitudinal Positive Reinforcement

(i) Effective width, $b_{ef} = 0.3 + 0.2 \times (0.85 \times 6) = 1.32$ m

(ii) Normalised moment, μ, is calculated: $\mu = \dfrac{M_{Sd}}{bd^2 f_{cd}} = \dfrac{132.7}{1.32 \times 0.47^2 \times 16.7 \times 10^3} = 0.027$

From Table 1[3] $\Rightarrow \omega = 0.028$; $\xi = 0.084$

(iii) Check if the neutral axis depth is within the section's flange:
x = 0.084 x 0.47 = 0.039 m (< 0.12m)

(iv) The area of steel is calculated: $A_s = \omega \dfrac{bdf_{cd}}{f_{yd}} = 0.028 \times \dfrac{1.32 \times 0.47 \times 16.7}{347.8} \cong 8.34\,\text{cm}^2$

From Table 2 $\Rightarrow 6\phi14$ (9.24cm^2)

(v) Detailing checks: The minimum area of reinforcement is given as 0.0015bd = 2.25 cm^2 $\Rightarrow$ OK
The maximum area of reinforcement is given as $0.04A_c$ = 60 cm^2 $\Rightarrow$ OK

Design of Longitudinal Negative Reinforcement

(i) Effective width, $b_{ef} = b_w = 0.3$m

(ii) Normalised moment, μ, is calculated: $\mu = \dfrac{M_{Sd}}{bd^2 f_{cd}} = \dfrac{187.85}{0.3 \times 0.47^2 \times 16.7 \times 10^3} = 0.170$

[3] CEB 1992

From Table 1[4] $\Rightarrow \omega = 0.192; \xi = 0.280$

(iii) The area of steel is calculated: $A_s = \omega \dfrac{bdf_{cd}}{f_{yd}} = 0.192 \times \dfrac{0.3 \times 0.47 \times 16.7}{347.8} \cong 13.00\,cm^2$

From Table 2 $\Rightarrow 7\phi16$ ($14.07cm^2$)

(iv) Detailing checks: The minimum area of reinforcement is given as 0.0015bd = 2.25 $cm^2 \Rightarrow$ OK
The maximum area of reinforcement is given as $0.04A_c$ = 60 $cm^2 \Rightarrow$ OK

1.1.6.7 Capacity Check for Seismic Loading of Beams

Actual moment capacity of positive reinforcement = $A_s f_{yd}(d-x/2)$
= 924 x 347.8 x (470-39/2) = 144.776 x 10^6 Nmm = 144.7 kNm
Actual moment capacity of negative reinforcement = $A_s f_{yd}(d-x/2)$
= 1407 x 347.8 x (470-131.6/2) = 197.8x10^6 Nmm = 197.8 kNm

The required moments for the seismic action-effects are 196 kNm at the support and 72.4 kNm at the span, which are lower than the actual capacities as calculated above.

[4] CEB 1992

1.1.6.8 Design of Transverse Reinforcement

The maximum shear force, V_{Sd} = 215kN, at the internal support of the left beam on the top floor is due to the vertical load case. The seismic load combination is only 144kN, so we design for the vertical load case, hence we use EC2. The design value of 215kN can be reduced to 215 - ρd = 183kN which corresponds to the shear force at a distance d from the support.

1.1.6.8.1 Check Maximum Design Force That Can Be Carried Without Crushing of Concrete

V_{Sd} = 183kN
V_{Rd2} = maximum design force that can be carried without crushing of concrete
$V_{Rd2} = 0.5\nu f_{cd} b_w \times 0.9d$ where $\nu = (0.7 - f_{ck}/200) = 0.575 \geq 0.5$
V_{Rd2} = 608.1 kN > 183kN ⇒ OK, there is no increase of dimensions required.

1.1.6.8.2 Check If Shear Reinforcement Is Required

V_{Rd1} = design shear of a member without shear reinforcement
$V_{Rd1} = [\tau_{Rd} k(1.2+40\rho_1)+0.15\sigma_{cp}]b_w d$

$\tau_{Rd} = 0.035 f_{ck}^{2/3} = 300\text{kPa}$

k = 1.6-d = 1.13, however, much of the bottom reinforcement will be curtailed before the supports (only 30% extends into the support) and thus does not contribute to shear resistance, so k will be taken as unity.

$$\rho = \frac{A_s}{b_w d} = \frac{7 \times \frac{\pi \times 16^2}{4} + 2 \times \frac{\pi \times 14^2}{4}}{300 \times 470} = 0.012 \left(\leq 0.02\right)$$

$\sigma_{cp} \approx 0$

So, V_{Rd1} = [300x1x(1.2+40x0.012)] x 0.3 x 0.47 =71 kN (<183kN)

Since $V_{Rd1} < V_{Sd}$ the beam requires shear reinforcement.

1.1.6.8.3 Design Required Shear Reinforcement

$V_{wd} = V_{Sd} - V_{Rd1} = 183 – 71 = 112\text{kN}$

The shear reinforcement is given by the following equation, where A_s and s are unknown:

$$\frac{A_{sw}}{s} \geq \frac{V_{wd}}{0.9 d f_{ywd}} = \frac{112}{0.9 \times 0.47 \times 191.3} = 1.38 \text{mm}^2/\text{mm}$$

Assuming a diameter of 10 mm, remembering that there are two links across the section, the maximum spacing is 114 mm. So, choose shear links, ϕ10@110mm

1.1.6.8.4 Check Code Limits

⇒ 1/5 x 608 = 121.6 kN; 2/3 x V_{Rd2} = 405.3kN
⇒ $1/5 V_{Rd2} < V_{Sd} < 2/3 V_{Rd1}$

The spacing of the transverse reinforcement should not exceed the lesser of the following three distances:

a) 12 x the minimum diameter of the longitudinal bars = 12 x 14 = 168mm
b) the least dimension of the column = 300mm
c) 300mm

⇒ OK

1.1.6.9 Column Design

The seismic load case is clearly the critical combination of actions for the columns. So this combination will be used for the design of the columns. Initial size of columns assumed from previous project as 500x400 mm.

1.1.6.9.1 M-N Pairs and Normalised Actions

The normalised actions are given by: $\mu = \dfrac{M_{Sd}}{bh^2 f_{cd}}$; $\nu = \dfrac{N_{Sd}}{bhf_{cd}}$

ω_{tot} is obtained from the M-N interaction diagrams

Internal Column

Storey	Seismic					Static				
	N (kN)	M (kNm)	μ	ν	ω_{tot}	N (kN)	M (kNm)	μ	ν	ω_{tot}
3	237	76.2	0.0456	0.057	0.080	419	74	0.0443	0.100	0
2	470	89.1	0.0534	0.113	0.030	830	47.7	0.0286	0.199	0
1	705	129	0.0772	0.169	0.060	1243	60.5	0.0362	0.298	0

External Column

Storey	Seismic					Static				
	N (kN)	M (kNm)	μ	ν	ω_{tot}	N (kN)	M (kNm)	μ	ν	ω_{tot}
3	121	126	0.0754	0.029	0.180	194	157	0.0940	0.046	0.16
2	256	106	0.0635	0.061	0.060	395	101	0.0605	0.095	0.02
1	394	134	0.0802	0.094	0.100	595	80.9	0.0484	0.143	0

The minimum total reinforcement ratio, ρ_{tot}, allowed by the code is 1%. This corresponds to a mechanical ratio, $\omega_{tot,min} = \rho_{tot}\,(f_{yd}/f_{cd}) = 0.01(347.8/16.7) = 0.208$, which is larger than all the values calculated.
Therefore the minimum requirement is used to determine the reinforcement areas for both exterior and interior columns:

$$A_{S1} = A_{S2} = \frac{1}{2}\omega_{tot} bh\left(f_{cd}/f_{yd}\right) = 0.5 \times 0.208 \times 2000 \times \left(16.7/347.8\right) = 10\,\text{cm}^2$$

Two 22 mm bars, one at each corner, and one intermediate 20 mm bar provide adequate reinforcement for the columns on each side. The total area is 27.8 cm^2 and the total reinforcement ratio is 1.4%.

In theory, the columns should be redesigned to give a more economical solution since they are being controlled by the minimum reinforcement requirements. However, this would result in the breaking of code limits so minimum requirements dominate in both the exterior and the interior columns.

Hence, one standard column design is used for both interior and exterior columns with minimum nominal steel reinforcement.

1.1.6.9.2 Capacity Design Considerations

The following calculations are based on the seismic load case 1.0G + 0.3Q + 1.0E.

Exterior Beam-Column Joints

For ductility class 'M', γ_{Rd} = 1.2

a) For direction '1' of seismic action,

the top of the beam is in tension

(i) actual beam moment capacity, $M_{Rd} = 182 \text{ x } (14.07/13.00) = 197 \text{ kNm}$

(ii) sum of moments ratio, $\alpha_{CD1} = \gamma_{Rd} \dfrac{\left|M^{l}_{R1}\right| + \left|M^{r}_{R1}\right|}{\left|M^{o}_{S1}\right| + \left|M^{u}_{S1}\right|}$

$$= 1.2 \text{x} \frac{0 + 197}{102 + 79.9} = 1.30$$

(iii) the moment reversal factor, $\delta_1 = \dfrac{\left|M^{r}_{S1} - M^{l}_{S1}\right|}{\left|M^{r}_{R1}\right| + \left|M^{l}_{R1}\right|} = \dfrac{|0 - 182|}{|0| + |197|} = 0.92$

(iv) capacity design requirement, $M_{Sd1,CD} = \left|1 + (\alpha_{CD,1} - 1)\delta_1\right| M_{Sd1}$

column over joint, $M^{o}_{Sd1,CD} = \left|1 + (1.30 - 1)0.92\right|(102) = 130.2$

axial load acting on column above joint, $N^{o} = -256\text{kN}$, hence $\nu = -0.08$

reinforcement ratio for one side of column, $\rho_1 = \dfrac{2\pi 22^2/4 + \pi 20^2/4}{400 \times 500} = 0.0054$

hence, $\omega_{tot} = 2\rho_1 (f_{yd}/f_{cd}) = 0.224$

giving from interaction diagram design charts, $\mu = 0.12$

thus, $M^{o}_{Rd} = \mu b h^2 f_{cd} = 0.12 \times 0.4 \times 0.5^2 \times 16.7 \times 10^3 = 200.4 > M^{o}_{CD} = 130.2$

thus there is no need to change the column reinforcement.

column under joint, $M^{u}_{Sd1,CD} = \left|1 + (1.30 - 1)0.92\right|(79.9) = 102.0$

axial load acting on column under joint, $N^{u} = -394\text{kN}$, hence $\nu = -0.12$

reinforcement ratio for one side of column, $\rho_1 = \dfrac{2\pi 22^2/4 + \pi 20^2/4}{400 \times 500} = 0.0054$

hence, $\omega_{tot} = 2\rho_1 (f_{yd}/f_{cd}) = 0.224$

giving from interaction diagram design charts, $\mu = 0.13$

thus, $M^{u}_{Rd} = \mu b h^2 f_{cd} = 0.13 \times 0.4 \times 0.5^2 \times 16.7 \times 10^3 = 217.1 > M^{u}_{CD} = 102.0$

thus there is no need to change the column reinforcement.

b) For direction '2' of seismic action,

the top of the beam is still in tension

(i) actual beam moment capacity, $M_{Rd} = 18.8 \text{ x } (14.07/13.00) = 20.3 \text{ kNm}$

(ii) sum of moments ratio, $\alpha_{CD2} = \gamma_{Rd} \dfrac{|M^{l}_{R2}| + |M^{r}_{R2}|}{|M^{o}_{S2}| + |M^{u}_{S2}|}$

$$= 1.2 \text{x} \frac{0 + 20.3}{10.5 + 8.38} = 1.29$$

(iii) the moment reversal factor, $\delta_2 = \dfrac{|M^{r}_{S2} - M^{l}_{S2}|}{|M^{r}_{R2}| + |M^{l}_{R2}|} = \dfrac{|0 + 18.8|}{|0| + |20.3|} = 0.93$

(iv) capacity design requirement, $M_{Sd2,CD} = |1 + (\alpha_{CD,2} - 1)\delta_2| M_{Sd2}$

column over joint, $M^{o}_{Sd2,CD} = |1 + (1.29 - 1)0.93|(10.5) = 13.3$

axial load acting on column above joint, $N^{o} = -187\text{kN}$, hence $\nu = -0.06$

reinforcement ratio for one side of column, $\rho_1 = \dfrac{2\pi 22^2/4 + \pi 20^2/4}{400 \times 500} = 0.0054$

hence, $\omega_{tot} = 2\rho_1 (f_{yd}/f_{cd}) = 0.224$

giving from interaction diagram design charts, $\mu = 0.115$

thus, $M^{o}_{Rd} = \mu b h^2 f_{cd} = 0.115 \times 0.4 \times 0.5^2 \times 16.7 \times 10^3 = 192.1 > M^{o}_{CD} = 13.3$

thus there is no need to change the column reinforcement.

column under joint, $M^{u}_{Sd2,CD} = |1 + (1.29 - 1)0.93|(8.4) = 10.7$

axial load acting on column under joint, $N^{u} = -272\text{kN}$, hence $\nu = -0.08$

reinforcement ratio for one side of column, $\rho_1 = \dfrac{2\pi 22^2/4 + \pi 20^2/4}{400 \times 500} = 0.0054$

hence, $\omega_{tot} = 2\rho_1 (f_{yd}/f_{cd}) = 0.224$

giving from interaction diagram design charts, $\mu = 0.12$

thus, $M^{u}_{Rd} = \mu b h^2 f_{cd} = 0.12 \times 0.4 \times 0.5^2 \times 16.7 \times 10^3 = 200.3 > M^{u}_{CD} = 10.7$

thus there is no need to change the column reinforcement.

Interior Beam-Column Joints

For ductility class 'M', $\gamma_{Rd} = 1.2$

a) For direction '1' of seismic action,

the top of the beam is in tension

(i) actual beam moment capacity, $M^{l}_{Rd} = 196 \text{ x } (14.07/13.00) = 212.1 \text{ kNm}$

$$M^{r}_{Rd} = 44 \text{ x } (14.07/13.00) = 47.6 \text{ kNm}$$

(ii) sum of moments ratio, $\alpha_{CD1} = \gamma_{Rd} \dfrac{|M^{l}_{R1}| + |M^{r}_{R1}|}{|M^{o}_{S1}| + |M^{u}_{S1}|}$

$$= 1.2 \text{x} \frac{212.1 + 47.6}{67 + 85.1} = 2.05$$

(iii) the moment reversal factor, $\delta_1 = \dfrac{|M^{r}_{S1} - M^{l}_{S1}|}{|M^{r}_{R1}| + |M^{l}_{R1}|} = \dfrac{|-44 - 196|}{|47.6| + |212.1|} = 0.92$

(iv) capacity design requirement, $M_{Sd1,CD} = |1 + (\alpha_{CD,1} - 1)\delta_1| M_{Sd1}$

column over joint, $M^{o}_{Sd1,CD} = |1 + (2.05 - 1)0.92|(85.1) = 167.3$

axial load acting on column above joint, $N^{o} = -470\text{kN}$, hence $\nu = -0.15$

reinforcement ratio for one side of column, $\rho_1 = \dfrac{2\pi 22^2/4 + \pi 20^2/4}{400 \times 500} = 0.0054$

hence, $\omega_{tot} = 2\rho_1 (f_{yd}/f_{cd}) = 0.224$

giving from interaction diagram design charts, $\mu = 0.15$

thus, $M^{o}_{Rd} = \mu b h^2 f_{cd} = 0.15 \times 0.4 \times 0.5^2 \times 16.7 \times 10^3 = 250.5 > M^{o}_{CD} = 167.3$

thus there is no need to change the column reinforcement.

column under joint, $M^{u}_{Sd1,CD} = |1 + (2.05 - 1)0.92|(67) = 131.7$

axial load acting on column under joint, $N^{u} = -705\text{kN}$, hence $\nu = -0.21$

reinforcement ratio for one side of column, $\rho_1 = \dfrac{2\pi 22^2/4 + \pi 20^2/4}{400 \times 500} = 0.0054$

hence, $\omega_{tot} = 2\rho_1 (f_{yd}/f_{cd}) = 0.224$

giving from interaction diagram design charts, $\mu = 0.17$

thus, $M^{u}_{Rd} = \mu b h^2 f_{cd} = 0.17 \times 0.4 \times 0.5^2 \times 16.7 \times 10^3 = 283.9 > M^{u}_{CD} = 131.7$

thus there is no need to change the column reinforcement.

b) For direction '2' of seismic action,

the top of the beam is still in tension

(i) actual beam moment capacity, $M^{l}_{Rd} = 186 \text{ x } (14.07/13.00) = 201.3 \text{ kNm}$

$$M^{r}_{Rd} = 44.2 \text{ x } (14.07/13.00) = 47.8 \text{ kNm}$$

(ii) sum of moments ratio, $\alpha_{CD2} = \gamma_{Rd} \dfrac{\left|M^{l}_{R2}\right| + \left|M^{r}_{R2}\right|}{\left|M^{o}_{S2}\right| + \left|M^{u}_{S2}\right|}$

$$= 1.2\text{x} \frac{201.3 + 47.8}{80.3 + 61.8} = 2.1$$

(iii) the moment reversal factor, $\delta_2 = \dfrac{\left|M^{r}_{S2} - M^{l}_{S2}\right|}{\left|M^{r}_{R2}\right| + \left|M^{l}_{R2}\right|} = \dfrac{|186 + 44.2|}{|201.3| + |47.8|} = 0.92$

(iv) capacity design requirement, $M_{Sd2,CD} = \left|1 + \left(\alpha_{CD,2} - 1\right)\delta_2\right| M_{Sd2}$

column over joint, $M^{o}_{Sd2,CD} = \left|1 + (2.1 - 1)0.92\right|(80.3) = 161.6$

axial load acting on column above joint, $N^{o} = -464\text{kN}$, hence $\nu = -0.14$

reinforcement ratio for one side of column, $\rho_1 = \dfrac{2\pi 22^2/4 + \pi 20^2/4}{400 \times 500} = 0.0054$

hence, $\omega_{tot} = 2\rho_1 \left(f_{yd}/f_{cd}\right) = 0.224$

giving from interaction diagram design charts, $\mu = 0.15$

thus, $M^{o}_{Rd} = \mu b h^2 f_{cd} = 0.15 \times 0.4 \times 0.5^2 \times 16.7 \times 10^3 = 250.5 > M^{o}_{CD} = 161.6$

thus there is no need to change the column reinforcement.

column under joint, $M^{u}_{Sd2,CD} = \left|1 + (2.1 - 1)0.92\right|(61.8) = 124.3$

axial load acting on column under joint, $N^{u} = -694\text{kN}$, hence $\nu = -0.21$

reinforcement ratio for one side of column, $\rho_1 = \dfrac{2\pi 22^2/4 + \pi 20^2/4}{400 \times 500} = 0.0054$

hence, $\omega_{tot} = 2\rho_1 \left(f_{yd}/f_{cd}\right) = 0.224$

giving from interaction diagram design charts, $\mu = 0.17$

thus, $M^{u}_{Rd} = \mu b h^2 f_{cd} = 0.17 \times 0.4 \times 0.5^2 \times 16.7 \times 10^3 = 283.8 > M^{u}_{CD} = 124.3$

thus there is no need to change the column reinforcement.

This capacity check ensures that the plastic hinge will occur in the beams before it occurs in the columns. Hence, the formation of the plastic hinge in the beam will lead to a stable mechanism of failure. As a result, the column reinforcements designed from static considerations need not be revised.

1.1.6.9.3 Transverse Reinforcement in Columns

As mentioned, only one standard column design is adopted for both internal and external columns. Hence, the design shear force including considerations of capacity design is the more critical of the two below.

$$\text{(i) Exterior column}: V_{Sd,CD} = \gamma_{Rd}\frac{M_{Rd}^{o} + M_{Rd}^{u}}{l_c} = 1.2 \times \frac{200.4 + 217.1}{3} = 167\text{kN}$$

$$\text{(i) Interior column}: V_{Sd,CD} = \gamma_{Rd}\frac{M_{Rd}^{o} + M_{Rd}^{u}}{l_c} = 1.2 \times \frac{250.5 + 283.9}{3} = 213.8\text{kN}$$

Hence the design shear force is 213.8kN.

Hoop Pattern

Since the maximum distance between adjacent longitudinal bars restrained by hoop bends is 200m for ductility class ‘M’, a double hoop pattern is used.

Minimum Diameter of Hoops

Minimum diameter of hoops,
$d_{bw} = \beta\ d_{bl,max}\ (f_{yld} / f_{ywd})$
$= 0.35 \times 22 \times (347.8/191.3)$
$= 14.0$ mm

Maximum Spacing of Hoops

$s_W = \min\{ b_c/3 ; 150\text{mm} ; 7d_{bl} \}$
$= \min\{ 340/3 ; 150\text{mm} ; 7\text{x}22\}$
$= 113$ mm
select spacing of 110 mm

Confinement Coefficients

$$\alpha_n = 1 - \frac{\sum b_i^2}{6A_0} = 1 - \frac{4 \times 0.159^2 + 4 \times 0.209^2}{6 \times 0.44 \times 0.34} = 0.693$$

$$\alpha_S = \left(1 - \frac{s_w}{2b_0}\right)^2 = \left(1 - \frac{110}{2 \times 340}\right)^2 = 0.703$$

$$\alpha = \alpha_n \alpha_S = 0.693 \times 0.703 = 0.487$$

For ductility class ‘M’ columns, the required mechanical ratio of confinement reinforcement is:

$$\omega_{Wd} \geq \frac{1}{\alpha}\left[k_0 \mu_\varphi \nu_d \left(\frac{f_{yd}}{E_S}\right)\left(0.35 A_g / A_c + 0.15\right) - 0.035\right]$$

$$\omega_{Wd} \geq \frac{1}{0.487}\left[60 \times 9 \times 0.21 \times 0.00174 \times \left(0.35 \times (0.4 \times 0.5)/(0.34 \times 0.44) + 0.15\right) - 0.035\right] = 0.178$$

Select hoops of 10 mm diameter at spacing s_w. The volumetric ratio is:

$$\rho_w = \frac{\text{volume of hoops}}{\text{volume of confined core}} = \frac{(2\times340+2\times440+4\times278)\times79}{340\times440\times s_w} = \frac{1.411}{s_w}$$

$$\omega_{Wd} = \rho_w (f_{yd}/f_{cd}) = \frac{1.411}{s_w}\times(347.8/16.7) = \frac{29.39}{s_w}$$

$$\omega_{Wd} \geq 0.178 \Rightarrow \text{maximum } s_w = 29.39/0.178 = 165\,\text{mm}$$

Thus the hoop spacing is dictated by maximum spacing of hoops above.
So choose $\phi 10@110$mm.

Check Maximum Design Force That Can Be Carried Without Crushing of Concrete

V_{Sd} = 213.8kN
V_{Rd2} = maximum design force that can be carried without crushing of concrete
$V_{Rd2} = 0.5\nu f_{cd} b_w \times 0.9d$ where $\nu = (0.7\text{-}f_{ck}/200) = 0.575 \geq 0.5$
V_{Rd2} = 760.5 kN > 213.8kN $\Rightarrow$ OK, there is no increase of dimensions required.

Check If Shear Reinforcement Is Required

V_{CD} = design shear of a member without shear reinforcement
$V_{CD} = [\tau_{Rd} k(1.2+40\rho_1)+0.15\sigma_{cp}]b_w d$

$\tau_{Rd} = 0.035 f_{ck}^{2/3}$ = 300kPa
k = 1.6-d = 1.6-0.46 = 1.14

$$\rho = \frac{A_s}{b_w d} = \frac{10.74}{40\times44} = 0.0061(\leq 0.02)$$

$$\sigma_{cp} = \frac{694}{0.4\times0.5} = 3470\,\text{kN/m}^2$$

So, V_{CD} = [300 x 1.14 x (1.2 + 40 x 0.0061) + 0.15 x 3470] x 0.4 x 0.5 =202.9 kN (<213.8 kN)

Shear Resistance

The shear resistance is given by the following equation;

$$V_{wd} = \frac{A_{sw}}{s}0.9df_{ywd} = \frac{(2+1.45)79}{110}\times0.9\times0.44\times191.3 = 187.7\text{kN}$$

Thus $V_{Rd} = V_{CD} + V_{wd}$ = 202.9 + 187.7 = 390.6

Outside Critical Limits

The spacing of the transverse reinforcement should not exceed the lesser of the following three distances:

d) 12 x the minimum diameter of the longitudinal bars = 12 x 20 = 240mm
e) the least dimension of the column = 300mm
f) 300mm

Therefore outside the critical regions the stirrup spacing can be increased to as much as 240 mm.

1.1.6.10 Detailing of Beams

1.1.6.10.1 Anchorage Length

The required anchorage length for bars and wires is given by:
$L_{b,net} = \alpha_a l_b (A_{s,req}/A_{s,prov}) \geq l_{b,min}$

At the support;
negative reinforcement anchorage length:
$L_{b,net}$ = 1.0 x (16/4 x (347.8/2.7)) x (14.07/13.0) = 557.7 mm

At the span;
Positive reinforcement anchorage length:
$L_{b,net}$ = 1.0 x (14/4 x (347.8/2.7)) x (9.24/8.34) = 499.5 mm

1.1.6.10.2 Curtailment of Support Reinforcement

1.1.7 Steel Design to EC8 for Earthquake Effects Based on Response Spectrum Analysis

Sufficient local ductility of members or parts of members in compression should be ensured by restricting the width-thickness ratio b/t according to the cross sectional classes as follows.

behaviour factor q	cross sectional class
$4 < q$	class 1
$2 < q \le 4$	class 2
$q \le 2$	class 3

Capacity design methodology must be employed.

Connections in dissipative zones should have sufficient over-strength to allow for yielding of the connected parts. Connections of dissipative parts made by means of butt welds or full penetration welds are deemed to satisfy the overstrength criterion. For fillet weld connections or bolted connections, the resistance of the connection must be factored 1.2 times the plastic moment capacity of the connected part. For bolted shear connections bearing failure should precede shear failure.

Moment resisting frames shall be designed so that plastic hinges form in the beams and not in the columns. This requirement is waived at the base of the frame, at the top floor of multistorey buildings and for one storey buildings. The beam to column joints shall have adequate overstrength to allow the plastic hinges to be formed in the beams.

Concentric braced frames shall be designed, so that yielding of the diagonals in tension will take place before yielding or buckling of the beams or columns and before failure of the connections.

Eccentric braced frames shall be designed so that beams are able to dissipate energy by the formation of plastic bending and/or plastic shear mechanisms. The rules are intended to ensure that yielding in the plastic hinges or shear panels of the beams will take place prior to any yielding or failure elsewhere.

1.1.8 Performance Based Seismic Analysis and Design

Recent damaging earthquakes in the United States (1989 Loma Prieta, 1994 Northridge) and abroad (1995 Kobe, Japan; 1999 Izmit, Turkey; and 1999 Chi-Chi, Taiwan) have led the public-at-large, public officials (local, state, and federal), and owners of structures (buildings, bridges, and infrastructure) to question the utility of modern seismic design codes. Many have argued that a higher level of public safety is required (despite the extremely low likelihood that an individual will die as a result of earthquake shaking). Partially in response to the damage wreaked by the Loma Prieta and Northridge earthquakes, the United States Federal Emergency Management Agency funded studies on performance based earthquake engineering: studies that resulted in FEMA 283 (Moehle and Whittaker, Eds), FEMA 273 and FEMA 274: two documents that have been replaced by FEMA 356.

When we talk about **performance based seismic design**, the definition of limit states for seismic design becomes extremely fundamental. They are defined as follows.

I. Level I Earthquake – Serviceability Limit State (Stiffness)
This limit state is so that cracking of concrete and limited steel yield due to small frequent earthquakes does not disrupt the function of the structure. **Stiffness governs the response of structures in low loads because strength yet to be reached.** Most operations and functions can resume immediately. Repair is required to restore some non-essential services. Damage is light. Structure is safe for immediate occupancy. Essential operations are protected.

II. Level II Earthquake – Damage Control Limit State (Strength and Distribution of Strength)
This limit state is so that the damage inflicted on the structure in the form of wide cracks in concrete and cover spalling and small permanent deformations in steel members due to the design earthquake is repairable. **Strength and its distribution govern the response as elastic limit reached.** Damage is moderate. Selected building systems, features or contents may be protected from damage. Life safety is generally protected. Structure is damaged but remains stable. Falling hazards remain secure. Repair possible.

III. Level III Earthquake – Survival Limit State (Ductility)
This limit state is so that the collapse of the structure and loss of life is avoided although heavy irreparable damage may occur due to the maximum credible earthquake. **Ductility governs the response.** Structural collapse is prevented. Nonstructural elements may fall. Repair generally not possible.

Performance based seismic analysis can be performed using the **multi-modal response spectrum (with no behaviour factor)** methodology, **random response** analysis or **deterministic transient** analysis.

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