Concepts of Linearised Buckling and Column Loading Analysis

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MAVERICK UNITED CONSULTING ENGINEERS

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1 Introduction

This paper describes the concepts of ***sway-sensitive*** and ***non-sway*** structures. Then a summary of **column loading analysis** and **effective lengths** is presented for ***braced*** and ***unbraced columns***. Finally, the concepts of **steel member design** in OASYS/GSA are presented.

2 Concepts of Sway-Sensitive and Non-Sway Structures

	Aspect		Full (Computerized) Method	Simplified / Hand Method
1	**Elemental Eigenvalue Buckling**	Flexural, shear and lateral torsional buckling		Flexural (Euler) buckling, $P_E = \frac{\pi^2 EI}{L_E^{\ 2}}$ Shear buckling, $N_S = G.A_S$ Lateral torsional buckling, $M_{LTB} = \frac{\pi}{L_E}\sqrt{EI_{MINOR}GJ}\sqrt{1+\frac{\pi^2 EI_w}{L_E^2 GJ}}$ Note Figure B.
2	**Building Eigenvalue Buckling**	Flexural buckling	Eigenvalue problem $\left[[K_E^A]+\lambda_{ECR}[K_G^{AKE}]\right]\{\varphi\}=\{0\}$ where K_G^{AKE} represents the geometric stiffness matrix which was calculated based on the small displacements obtained by solving the system (with the collapsing load) with stiffness K_E^A. Note Figure A.	Horne's approximate bifurcation analysis of frames estimate of critical load factor, $\lambda_{ECR} = 0.9(\Sigma NHF/\Sigma V)/(\Delta\delta/H)$ In perfect analogy, the code computes λ_{ECR} for symmetrical multi-storey buildings based on the deflection due to the NHF of 0.5% of the factored vertical (1.4 dead + 1.6 live) load (BS 5950-Part 1:2000 cl.2.4.2.6) or 1.0% of the factored vertical (1.4 dead) load (BS 8110), applied at the same level as follows $\lambda_{ECR} = \frac{\sum NHF}{\sum V} / \frac{\Delta\delta}{H} = \frac{\sum NHF}{\sum V}.\frac{H}{\Delta\delta}$ $\lambda_{ECR} = 0.5\%.\frac{H}{\Delta\delta} = \frac{1}{200}\frac{H}{\Delta\delta}$ (steel structures) $\lambda_{ECR} = 1.0\%.\frac{H}{\Delta\delta} = \frac{1}{100}\frac{H}{\Delta\delta}$ (concrete structures) where $H/\Delta\delta$ is the value of the storey height divided by the storey drift for any storey in the building.

3	**Elemental P-Δ Based Buckling**	Flexural buckling, lateral torsional buckling		Note there are two effects under consideration, firstly the **second order effects** and secondly the **imperfections**. BS 5950-Part 1:2000 (cl.4.7.4) flexural (Perry-Robertson) buckling with imperfections and residual stresses, $\frac{P_{PR}}{A} + \frac{M}{Z}\left(\frac{1}{1-P_{PR}/P_E}\right) = \sigma_y$ BS 5950-Part 1:2000 (cl.4.3) lateral torsional (Perry-Robertson) buckling with imperfections and residual stresses, $M_{LTB} = S_x \cdot \left(\frac{p_E p_y}{\phi_{LT} + \left(\phi_{LT}^2 - p_E p_y\right)^{0.5}}\right)$ Note Figure B.
4	**Building P-Δ Based Buckling**	Flexural buckling, lateral torsional buckling	Note there are two effects under consideration, firstly the **second order effects** and secondly the **imperfections**. • If $\lambda_{ECR} > 10$ then P-Δ effects are insignificant (non-sway) and can be neglected – Perform linear analysis. • If $4 < \lambda_{ECR} < 10$, P-Δ effects should be incorporated (sway-sensitive) – Perform P-Δ analysis. $\left[K_E^A + K_G^{AKE}\right]\{U\}$ $= \{P\} + \{\text{Fixed End Forces}\} - \left[K_G^{AKE}\right]\{U_0\}$ where the geometric stiffness K_G^{AKE} caters for the **second order effects** and the term $[K_G^{AKE}].\{U_0\}$ accounts for the **imperfections**. • If $\lambda_{ECR} < 4$, a second order nonlinear analysis should be undertaken. This effectively implies that the use of the P-Δ approach to predict the buckling load factor is not possible as the method is not accurate when $\lambda_{ECR} < 4$. Note Figure A.	Note there are two effects under consideration, firstly the **second order effects** and secondly the **imperfections**. • If $\lambda_{ECR} > 10$ then P-Δ effects are insignificant (non-sway) and can be neglected – Perform linear analysis. • If $4 < \lambda_{ECR} < 10$, P-Δ effects should be incorporated (sway-sensitive) *note – Perform P-Δ analysis. Lateral loads (wind, earthquake) within the combination cases need to be manually enhanced based on the amplified sway factor, m to cater for the **second order effects**, $m = \frac{\lambda_{ECR}}{\lambda_{ECR} - 1}$ The **imperfections** in turn are accounted for by the NHF combination case, 1.4DL+1.4SDL+1.6LL+1.6Snow ± 1.0NHF • If $\lambda_{ECR} < 4$, a second order nonlinear analysis should be undertaken. This effectively implies that the use of the P-Δ approach to predict the buckling load factor is not possible as the method is not accurate when $\lambda_{ECR} < 4$.

*Note: BS 5950 states that a column may be considered as non-sway in a given plane if the elastic buckling load factor, λ_{ECR} is of a value greater than 10.0 (cl.2.4.2.6).

The American Concrete Code ACI318M-08 defines sway-sensitive and non-sway structures based on the stability coefficient, Q as follows: -

10.10.5.2 — It also shall be permitted to assume a story within a structure is nonsway if:

$$Q = \frac{\Sigma P_u \Delta_o}{V_{us} \ell_c} \leq 0.05 \qquad (10\text{-}10)$$

where ΣP_u and V_{us} are the total factored vertical load and the horizontal story shear, respectively, in the story being evaluated, and Δ_o is the first-order relative lateral deflection between the top and the bottom of that story due to V_{us}.

This ACI effectively states that the structure is considered as non-sway if $Q \leq 0.05$ i.e. if $\lambda_{ECR} \geq 20$ (c.f. the British BS 5950 and European EC2 code which sets the criteria at $\lambda_{ECR} \geq 10$).

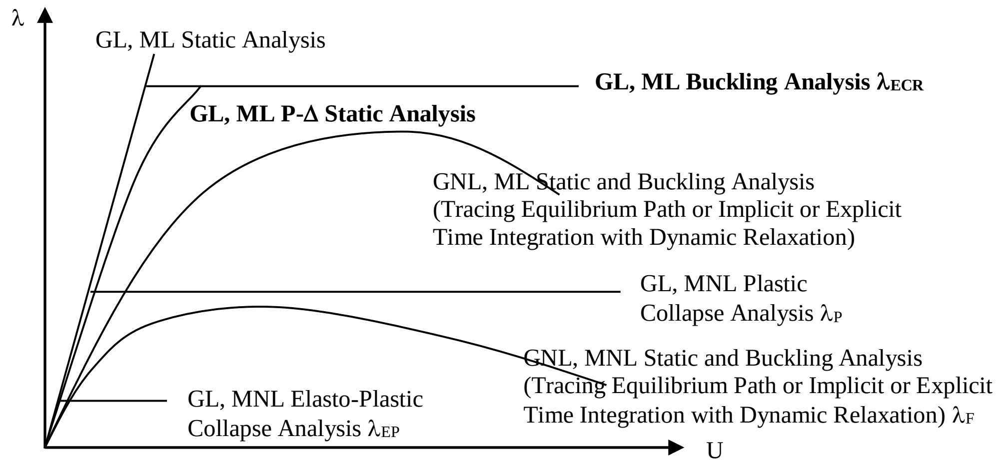


Figure A

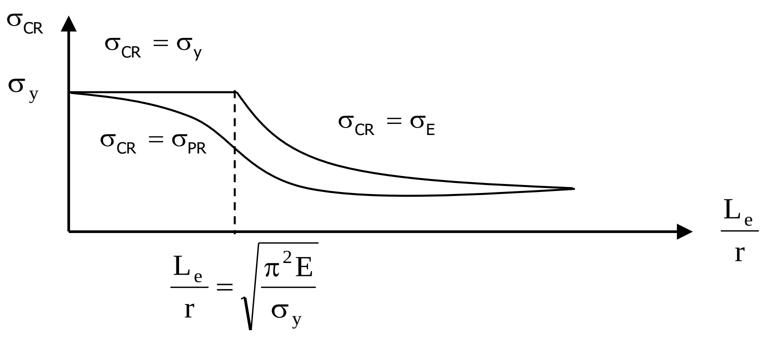


Figure B

3 Column Loading Analysis and Column Effective Lengths

The definitions of braced and unbraced members according to EC2 are as follows: -

- **Braced members or systems**: structural members or subsystems, which in analysis and design are assumed not to contribute to the overall horizontal stability of a structure.
- **Unbraced members or systems**: structural members or subsystems, which in analysis and design are assumed to contribute to the overall horizontal stability of a structure. Note that **unbraced members** are also appropriately known as **bracing members**.

The equivalent BS 8110 definitions are as follows: -

- A column may be considered **braced** in a given plane if lateral stability to the structure as a whole is provided by walls or bracing or buttressing designed to resist all lateral forces in that plane. It should otherwise be considered as **unbraced** (cl.3.8.1.5).
- **An unbraced wall** is a wall providing its own lateral stability (cl.1.3.4.2).
- **A braced wall** is a wall where the reactions to lateral forces are provided by lateral supports (cl.1.3.4.3).

COLUMN LOADING ANALYSIS EXECUTIVE SUMMARY

BRACED COLUMNS
LATERAL STABILITY SYSTEM = BRACING / SHEAR WALL

CONCRETE COLUMN (BS 8110)

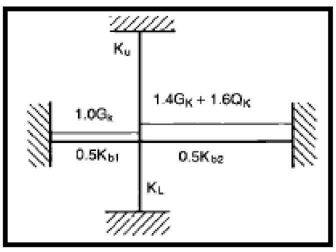
1. Axial force from loading tributary (all floors), ΣN

2. Bending moment, M from **MAX** of
   (i) imperfection eccentricity bending moment, $M_1 = \Sigma N \cdot \text{MIN}(0.05h, 20mm)$
   (ii) primary + slenderness bending moment, $m.M_2 + M_3$ where $M_2 = MF_U$ or MF_L

$$\boxed{MF_U = M_{es}\frac{K_u}{K_L + K_u + 0.5K_{b1} + 0.5K_{b2}}}$$

$$\boxed{MF_L = M_{es}\frac{K_L}{K_L + K_u + 0.5K_{b1} + 0.5K_{b2}}}$$

$M_{es} = \text{ABS}(M_{FEM,\,b2} - M_{FEM,\,b1})$
$M_{FEM,b1} = \omega_{Gk}L_{b1}^2/12$
$M_{FEM,\,b2} = \omega_{1.4Gk+1.6Qk}L_{b2}^2/12$
$K_{b1} = 4EI_{b1}/L_{b1}$
$K_{b2} = 4EI_{b2}/L_{b2}$
$K_u = 4EI_u/L_u$
$K_l = 4EI_l/L_l$



3. Shear force, V from $M / (H_{storey} / 2)$

STEEL COLUMN (BS 5950)

1. Axial force from loading tributary (all floors), ΣN

2. Bending moment, M from **SUMMATION** (however mutually exclusive) of
   (i) eccentricity bending moment, $M_1 = \frac{1}{2} . [\{V_{b2} - V_{b1}\} . \{(D \text{ or } t) / 2 + n_{ecc}\}]$
   $V_{b1} = \omega_{Gk}L_{b1}/2$
   $V_{b2} = \omega_{1.4Gk+1.6Qk}L_{b2}/2$
   where $n_{ecc} \geq 100mm$.

   Note should the bolt group holes be of standard clearance, and thus allowing the assumption of the pinned location at the face of the column, the minimum stipulated n_{ecc} of 100mm (*cl.4.7.7 BS 5950*) would apply. Should the bolt group holes not be of standard clearance, the pinned location should then be assumed at the centroid of the bolt group, thus resulting in the possibility of the distance from the column face to the centroid of the bolt group, n_{ecc} being greater than 100mm.

   (ii) primary bending moment, $m.M_2$
   Note M_2 only exists for continuous beam to column connections with the evaluation procedure as that for concrete columns; for simple (pinned) beam to column connections, $M_2 = 0.0$.

3. Shear force, V from $M / (H_{storey} / 2)$

UNBRACED COLUMNS LATERAL STABILITY SYSTEM = MOMENT FRAME	1. Axial force from loading tributary (all floors), $\Sigma N + N_{EXT} = \pm [\omega_{MF} . H_T^2 / 2] / (n . L_{MF})$ where L_{MF} is the bay span and n is the no. of bays. 2. Bending moment, M from **MAX** of (i) imperfection eccentricity bending moment, $M_1 = \Sigma N$. MIN (0.05h, 20mm) (ii) primary + slenderness bending moment, $m.M_2 + M_3$ where $M_2 = M_{INT}$ or M_{EXT} $\omega_{NHF,T}$ (kN/m) = 1.5% . [unfactored storey dead load] / H_{storey} $\omega_{WIND,T}$ (kN/m) = wind pressure (kPa) . building width (m) $\omega_{MF} = MAX (\omega_{NHF,T}, 1.4\ \omega_{WIND,T}) . B_{MF} / B_T$ where B_{MF} is the moment frame lateral load tributary width and B_T is the total building width. base shear, $\Sigma V = H_T . \omega_{MF}$ $V_{INT} = \Sigma V$ / no. of bays, n $V_{EXT} = ½\ V_{INT}$ $M_{INT/EXT} = ½\ V_{INT/EXT} . H_{storey}$ where V_{INT}, V_{EXT}, M_{INT} and M_{EXT} are internal and external column shears and moments. Note it is assumed the horizontal loading effects produce greater bending moments than that which is produced by the vertical sub-frame effect, failing which, clearly the latter should be adopted. 3. Shear force, V from M / (H_{storey} / 2)	1. Axial force from loading tributary (all floors), $\Sigma N + N_{EXT} = \pm [\omega_{MF} . H_T^2 / 2] / (n . L_{MF})$ where L_{MF} is the bay span and n is the no. of bays. 2. Bending moment, M from *(i) note the eccentricity bending moment, $M_1 = 0.0$.* (ii) primary bending moment, $m.M_2$ Note evaluation procedure for M_2 is as that for concrete columns with the exception of the definition of the notional horizontal force. $\omega_{NHF,T}$ (kN/m) = 0.5% . [factored storey dead and live load] / H_{storey} Note it is assumed the horizontal loading effects produce greater bending moments than that which is produced by the vertical sub-frame effect, failing which, clearly the latter should be adopted. 3. Shear force, V from M / (H_{storey} / 2)

The above presents the effects on the columns. Another major consideration is the **effective lengths** of the columns. BS 8110 gives effective length factors for braced columns (within a shear wall lateral stability system) in Table 3.19 and effective length factors for unbraced columns (within a moment frame lateral stability system) in Table 3.20.

Table 3.19 — Values of β for braced columns

End condition at top	End condition at bottom		
	1	2	3
1	0.75	0.80	0.90
2	0.80	0.85	0.95
3	0.90	0.95	1.00

Table 3.20 — Values of β for unbraced columns

End condition at top	End condition at bottom		
	1	2	3
1	1.2	1.3	1.6
2	1.3	1.5	1.8
3	1.6	1.8	—
4	2.2	—	—

3.8.1.6.2 *End conditions*

The four end conditions are as follows.

a) *Condition 1.* The end of the column is connected monolithically to beams on either side which are at least as deep as the overall dimension of the column in the plane considered. Where the column is connected to a foundation structure, this should be of a form specifically designed to carry moment.

b) *Condition 2.* The end of the column is connected monolithically to beams or slabs on either side which are shallower than the overall dimension of the column in the plane considered.

c) *Condition 3.* The end of the column is connected to members which, while not specifically designed to provide restraint to rotation of the column will, nevertheless, provide some nominal restraint.

d) *Condition 4.* The end of the column is unrestrained against both lateral movement and rotation (e.g. the free end of a cantilever column in an unbraced structure).

This is **in addition** to sway-sensitivity checks which are required and shall result in the need for the application of the amplified sway factor, m to lateral loads should the elastic buckling load factor, λ_{ECR} be of a value between 4.0 and 10.0. This is suggested even though BS 8110 cl.3.8.3.7 already specifies additional bending moments for **unbraced slender** columns at the **ends** of the member as if simulating global structure (and also local member for that matter) P-Δ effects. Note that BS 8110 cl.3.8.3.2 on the other hand specifies for **braced slender** columns additional bending moments at the **middle** of the member signifying local member P-Δ effects only. Thus, in effect, BS 8110 does not specify any global P-Δ effects for **braced slender, braced stocky (short)** and **unbraced stocky (short)** columns. In order to simulate this, the additional application of the amplified sway factor, m to lateral loads is hence suggested even though not explicitly required by the BS 8110 code. Note that EC2 on the other hand explicitly specifies the need for the application of the amplified sway factor, m to lateral loads should the elastic buckling load factor, λ_{ECR} be of a value between 4.0 and 10.0. The EC2 code further distinguishes global structure and local member P-Δ effects by specifying that **slender** columns are subject to additional bending moments at the **middle** of the member and thus simulating local member P-Δ effects irrespective of whether the column is braced or unbraced (although the effective lengths of braced and unbraced columns do indeed differ with the latter being longer).

In steel structures on the other hand, BS 5950 cl.5.6.4 gives two methods by which sway-sensitive frames (i.e. that with $4.0 < \lambda_{ECR} < 10.0$) may be analysed. The first method is the **effective length method** whereby the sway mode effective lengths of Table 22 and Annex E are employed without consideration of the amplified sway factor, m on lateral loads. The second method is the **amplified sway method** whereby the non-sway mode effective lengths of Table 22 and Annex E is employed with consideration of the amplified sway factor, m being applied to lateral loads should the elastic buckling load factor, λ_{ECR} be of a value between 4.0 and 10.0.

Table 22 — Nominal effective length L_E for a compression member[a]

a) non-sway mode			
Restraint (in the plane under consideration) by other parts of the structure			L_E
Effectively held in position at both ends	Effectively restrained in direction at both ends		0.7L
	Partially restrained in direction at both ends		0.85L
	Restrained in direction at one end		0.85L
	Not restrained in direction at either end		1.0L
b) sway mode			
One end	Other end		L_E
Effectively held in position and restrained in direction	Not held in position	Effectively restrained in direction	1.2L
		Partially restrained in direction	1.5L
		Not restrained in direction	2.0L

[a] Excluding angle, channel or T-section struts designed in accordance with 4.7.10.

A summary of the column effective length condition classification is thus presented: -

Structural Material and Stability System	Sway Sensitivity Scenario and Application of P-Δ Amplified Sway Factor [# Note 1]	Method 1: Column Effective Length Condition Adopted for Column Design (Code Based) [# Note 2]	Method 2: Column Effective Length Condition Adopted for Column Design (Moment Ratio Based) [# Note 3]	Method 3: Column Effective Length Condition Adopted for Column Design (Sway Sensitivity Based) [# Note 4]
RC Moment Frame BS 8110	λ_{ECR} < 10.0 (sway-sensitive) $\therefore$ m > 1.0	Unbraced (T.3.20)	Unbraced (T.3.20)	Unbraced (T.3.20)
	λ_{ECR} > 10.0 (non-sway) $\therefore$ m = 1.0	Unbraced (T.3.20)	Unbraced (T.3.20)	Braced (T.3.19)
RC Shear Wall BS 8110	λ_{ECR} < 10.0 (sway-sensitive) $\therefore$ m > 1.0	Braced (T.3.19)	Braced (T.3.19) or Unbraced (T.3.20)	Unbraced (T.3.20)
	λ_{ECR} > 10.0 (non-sway) $\therefore$ m = 1.0	Braced (T.3.19)	Braced (T.3.19) or Unbraced (T.3.20)	Braced (T.3.19)
Steel Moment Frame BS 5950 [# Note 5]	λ_{ECR} < 10.0 (sway-sensitive) $\therefore$ m > 1.0	Non-sway mode (T.22a)	Non-sway mode (T.22a)	Non-sway mode (T.22a)
	λ_{ECR} > 10.0 (non-sway) $\therefore$ m = 1.0	Non-sway mode (T.22a)	Non-sway mode (T.22a)	Non-sway mode (T.22a)
Steel Shear Wall BS 5950 [# Note 5]	λ_{ECR} < 10.0 (sway-sensitive) $\therefore$ m > 1.0	Non-sway mode (T.22a)	Non-sway mode (T.22a)	Non-sway mode (T.22a)
	λ_{ECR} > 10.0 (non-sway) $\therefore$ m = 1.0	Non-sway mode (T.22a)	Non-sway mode (T.22a)	Non-sway mode (T.22a)

#Note 1: The sway-sensitivity checks shall result in the need for the application of the amplified sway factor, m to lateral loads should the elastic buckling load factor, λ_{ECR} be of a value between 4.0 and 10.0, for all the methods of column bracing classification evaluation.
#Note 2: The **code based** column bracing classification evaluation for **RC structures** refers to the concept that structures with shear wall stability elements will result in braced columns and structures with moment frame stability elements (i.e. without shear wall stability elements) will result in unbraced columns.
#Note 3: The **moment ratio based** column bracing classification evaluation for **RC structures** refers to the concept that structures with stiff shear wall stability elements such that the axial forces (from global moment frame bending moments) and local bending moments (from local shear forces) in the columns (which inevitably form part of a moment frame stability system) from lateral loads only are negligible, will result in braced columns. Conversely, structures with flexible shear wall stability elements such that the axial forces (from global moment frame bending moments) and local bending moments (from local shear forces) in the columns (which inevitably form part of a moment frame stability system) from lateral loads only are not negligible, will result in unbraced columns. Finally, structures with only moment frame stability elements (i.e. without shear wall stability elements) will result in unbraced columns.
#Note 4: The **sway sensitivity based** column bracing classification evaluation for **RC structures** refers to the concept that non-sway structures will result in braced columns and sway-sensitive structures will result in unbraced columns.
#Note 5: For **steel structures**, all members may adopt the non-sway mode effective lengths of Table 22 and Annex E of BS 5950 with consideration of the amplified sway factor, m already being applied to lateral loads to cater for the global P-Δ effects.

Structural Material and Stability System	Remark on the Interpretation of Column Effective Length Condition Classification		
	Code Based	Moment Ratio Based	Sway Sensitivity Based
RC Moment Frame BS 8110	OK	OK	OK
	Conservative	Conservative	Less conservative
RC Shear Wall BS 8110	Less conservative	Less conservative or conservative	Conservative
	OK	OK or conservative	OK
Steel Moment Frame BS 5950	OK	OK	OK
	OK	OK	OK
Steel Shear Wall	OK	OK	OK

BS 5950	OK	OK	OK

4 Concepts of Steel Member Design in OASYS/GSA

1. Provide **ONE (compression) flange restrained** member **between points of contraflexure** for single or multiple (finite) elements when: -
   (a) modelling a composite beam in sagging

2. Provide **ONE simply-supported** member **between points of contraflexure** for single or multiple (finite) elements when: -
   (a) modelling a composite beam in hogging

3. Provide **ONE simply-supported** member **between supports** for single or multiple (finite) elements when: -
   (a) modelling a member totally unrestrained (at its compression flange or web) within its span but fully restrained at the supports

4. Provide **ONE simply-supported** member **between intermediate lateral restraints** for single or multiple (finite) elements when: -
   (a) modelling a member totally unrestrained (at its compression flange or web) between intermediate lateral restraints but fully restrained at the intermediate lateral restraints

5. Provide **ONE simply-supported** member **between supports** for single or multiple (finite) elements when: -
   (a) modelling a member where the restraints (at its compression flange or web) provided by the supports are uncertain as to their effectiveness, but restraints are indeed desired, in a model that accounts for buckling, i.e.
      i. static analysis for a model with no buckling effects i.e. if modal buckling fundamental load factor, $\lambda_{ECR} \geq 10.0$.
      ii. p-delta static analysis for a model with moderate buckling effects i.e. if modal buckling fundamental load factor, $\lambda_{ECR} \geq 4.0$.

6. Provide **ONE simply-supported** member **between intermediate lateral restraints** for single or multiple (finite) elements when: -
   (a) modelling a member where the restraints (at its compression flange or web) provided by intermediate lateral restraints are uncertain as to their effectiveness, but restraints are indeed desired, in a model that accounts for buckling, i.e.
      i static analysis for a model with no buckling effects i.e. if modal buckling fundamental load factor, $\lambda_{ECR} \geq 10.0$.
      ii. p-delta static analysis for a model with moderate buckling effects i.e. if modal buckling fundamental load factor, $\lambda_{ECR} \geq 4.0$.

7. Provide **ONE cantilever** member **between points of contraflexure** (i.e. the cantilever span) for single or multiple (finite) elements when: -
   (a) modelling a cantilever